Structural-temporal theory of fracture as a multiscale process

Yu.V. Petrov*[1,2], A.A. Gruzdkov[3], and V.A. Bratov[1,2]

[1] Institute of Problems of Mechanical Engineering RAS, St. Petersburg, 199178, Russia
[2] St. Petersburg State University, St. Petersburg, 199034, Russia
[3] St. Petersburg State Institute of Technology (Technical University), St. Petersburg, 190013, Russia

The paper reports on a structural-temporal approach to analysis of multiscale fracture of solids. A practical procedure is proposed for estimating the strength characteristics of material on one scale from test data on another scale.

Keywords: structural fracture mechanics, dynamic strength, structural-temporal approach, incubation time, scale hierarchy

DOI: 10.1134/S1029959912020117

1. Introduction

In the last decades, numerous research data on the strength of materials and conditions of their fracture have revealed serious discrepancies in both values of the strength characteristics of many materials and their qualitative dependences on the loading conditions. One of the major causes for the discrepancies is that the same term “fracture” is applied to different events preceded by different physical processes often of different scales.

The significance of understanding the fracture of solids not as a critical event but as a process evolving in time on many scales was eventually realized by many scientists and the hierarchy of different scales of fracture became much addressed in the literature [1–5]. Fracture is not a simple process but is a set of concurrent processes differing in characteristic linear dimension, activation energy, threshold stress, and relaxation time. This circumstance owes in many respects to difficulties that arise in attempts to relate the macroscopic parameters of engineering models and the microscopic parameters of corresponding processes. An important aspect is the scale effect involved in the dependence of strength characteristics on the dimensions of a construction. Tests by GOST are often conducted on laboratory specimens; however, the parameters of a material measured under these conditions are generally inapplicable to predict the strength of microobjects as well as large-scale structures made of the material. Ignorance of this fact can lead to false calculations and even to technogenic catastrophes.

The difficulty and, in many cases, the impossibility to perform consistent tests on many scales generates a need to determine the strength characteristics of one scale from test data of another scale. By now, there is no appropriate procedure suitable for direct use in engineering practice. Noteworthy also is the absence of commonly accepted concepts on what exactly we should consider as different scales of fracture.

Of fundamental importance, in our viewpoint, are the following questions: how to define a particular scale; what tests are correct for a given scale; and whether a relationship between strength parameters on different scales is possible to establish. It is these questions which are examined in the present paper.

2. Structural fracture mechanics

The high degree of universality of equations of continuum mechanics has its reverse side — the minimum number of parameters gives no way of describing the variety of properties displayed by different materials. Really, the motion of a linearly elastic isotropic medium is described by the Lamé equations:

$$\rho\ddot{\mathbf{u}} = (\lambda+\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{u} + \mu\Delta\mathbf{u},$$

where $\mathbf{u}$ is the displacement vector; ρ is density; λ, μ are Lamé constants. The velocity of a longitudinal wave responsible for a change in volume is equal to $c_1 =$

* *Corresponding author*
Prof. Yurii V. Petrov, e-mail: yp@yp1004.spb.edu

$=\sqrt{(\lambda+2\mu)/\rho},$ and that of a transverse wave is equal to $c_2=\sqrt{\mu/\rho}$. Thus, the description of the whole variety of materials differing in structure is reduced to two parameters c_1 and c_2 and, in view of mechanical similarity, to only one parameter — the longitudinal to traverse wave velocity ratio.

Adequate description of strength properties requires introduction of additional characteristics that allow for structural peculiarities of examined materials. The main requirements on these characteristics are the following: they should offer sufficient universality, i.e., should not depend on the peculiarities of specific loading conditions, their number should be small, and they should admit of experimental determination. In practical terms, one of the main characteristics is the critical stress or the ultimate strength. The condition of conservation of integrity, in this case, has the form:

$$\sigma(t)\le\sigma_c, \tag{1}$$

where $\sigma(t)$ is the applied stress; σ_c is the ultimate strength. However, the classical criterion of critical stress is found inapplicable to problems with high stress gradients (e.g., with a stress concentrator) or sharp load differences (with a high stress rate).

For a solid with a crack, the fracture condition (the crack growth condition) is taken to be the attainment of a critical stress intensity factor:

$$K_{\rm I}(t)\le K_{\rm Ic}, \tag{2}$$

where $K_{\rm I}(t)$ is the current stress intensity factor; $K_{\rm Ic}$ is the critical stress intensity factor. Criterion (1) is applied to defect-free solids, and criterion (2), to solids with a macroscopic defect like a crack.

The presence of two radically different strength characteristics puts several questions: what defect size should be considered macroscopic, what fracture condition should be used in problems with a stress singularity distinct from that for a plane crack (a solid with an angular notch), etc. It is also evident that given the characteristics $K_{\rm Ic}$ and σ_c, it is possible to obtain the material constant of length dimension:

$$d=\frac{2}{\pi}\frac{K_{\rm Ic}^2}{\sigma_c^2}. \tag{3}$$

The introduction of the characteristic linear size as a strength parameter of material provides the unified strength criterion of solids which transforms to (1) or (2) in its limiting cases:

$$\frac{1}{d}\int\limits_{x-d}^{x}\sigma(s,t)\,{\rm d}s\le\sigma_c. \tag{4}$$

Criterion (4) was proposed by Novozhilov in [6], and earlier by Neiber. After unsuccessful attempts to relate the parameter d to the characteristic structural sizes of material (interatomic distances, grain size, etc.), the conclusion was made that this parameter is a characteristic of fracture as such, more precisely, of that scale on which consideration is taken. Criterion (4) reflects the discrete character of fracture.

A similar problem arises when we consider dynamic strength of solids, e.g., in simulation of cleavage failure. If the time of external action is short enough, criterion (1) becomes invalid: for a short time, a material is capable of withstanding stresses much higher than the static ultimate strength σ_c without fracture. It is likely that this effect shows up most vividly in the presence of two branches — static and dynamic — of a fatigue life diagram, i.e., of the dependence of the time before fracture on the amplitude of applied stress [7–9] (Fig. 1). For the static branch, the time before fracture can vary widely, while the fracture occurs at one stress level corresponding to static ultimate strength. For the dynamic branch, a considerable change in stress amplitude fails to greatly shorten the time before fracture and in some cases no fracture is detected at stresses much higher than static ultimate strength.

The dynamic increase in limiting stress can be determined qualitatively by the critical impulse criterion proposed, e.g., in [10],

$$\int\limits_0^{t_*}\sigma(t)\,{\rm d}t\le C, \tag{5}$$

where t_* is the time before fracture. However, criterion (5), though being qualitatively consistent with experimental observations, contradicts quasistatic criterion (1), and therefore, it is applicable only qualitatively to rather rapid loading, i.e., this criterion fails to describe both the static branch of the time dependence of strength and the transition to it. Moreover, the position of the dynamic branch on the time dependence of strength is uncertain and it is thus unclear what loading pulse duration should be considered sufficiently short to apply criterion (5); what loading should be considered slow enough to apply criterion (1); what criterion should be used in an intermediate case.

The above problems can be solved, if we introduce, along with the spatial structure specified by the parameter d, a structural parameter on the time axis [11–13] — the incubation time τ. The unified fracture criterion can be written in the form:

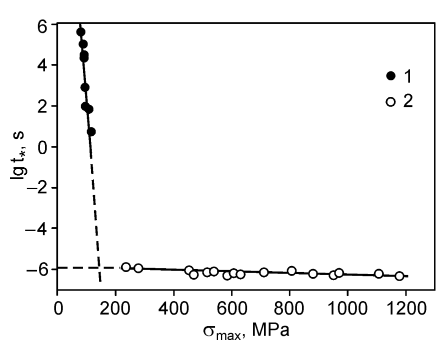

Fig. 1. Static and dynamic branches of cleavage strength for aluminum from the data of [7]. The dependence of the time before fracture on the stress amplitude: quasistatic tests (*1*), pulsed tension (*2*)

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(\xi)\,d\xi<\sigma_c. \quad (6)$$

Criterion (6) defines in a unified manner both the dynamic and static branches [11].

The approach based on the incubation time was found efficient in simulation of a wide range of physical phenomena: yielding of metals, cavitation of liquids, electrical breakdown, etc. Related data are generalized in many papers on the structural-temporal approach, in particular, in [13].

It is significant that in the framework of this approach, the strength of a material is defined by three characteristics: static ultimate strength σ_c, characteristic linear size d, and incubation time τ which are experimentally determined parameters. The ultimate strength can be determined directly from data of quasistatic tests; the characteristic linear size, from comparison of data for defect-free and cracked specimens; and the incubation time, from comparison of strengths corresponding to two different loading rates. In the general case, the fracture criterion can be represented in the form [11, 12]:

$$\frac{1}{\tau}\frac{1}{d}\int_{x-d}^{x}\int_{t-\tau}^{t}\sigma(x,t)\,dx dt\geq\sigma_c. \quad (7)$$

3. Scales of fracture

It is sometimes omitted that estimation of strength characteristics of a material is inevitably related to the question of what fact should be considered as a fact of fracture. In quasistatic tests, fracture traditionally means complete separation of a specimen into parts (fragmentation). At the same time, in cleavage tests, the instant of fracture is normally determined by a jump on the free surface velocity diagram of a specimen, which corresponds to the formation of a defect inside the specimen. Because the formation of a large defect is preceded by the formation, growth, and merging of smaller defects, there is an inevitable question as to the critical defect size from which the formation of a defect can be considered as macroscopic fracture. This problem is associated primarily with characteristics of employed measuring equipment. So in the limiting case, fracture can be taken as rupture of one elementary bond.

Calculations show, for example, that for polymethylmethacrylat under pulsed loading in the microsecond range, a considerable change in the signal of an interferometer measuring the free surface velocity can produce subsurface cracks of size 100–200 μm. In this context, it is natural to expect a noticeable discrepancy in the dynamic branch position in comparison of data on cleavage tests of rods and plates. This discrepancy did take place in processing of the experimental data of [8] and their comparison with the data of [9] and other works (Fig. 2).

In the structural-temporal approach described above, fracture on a given scale is understood as the formation of a defect of characteristic linear size d. The appearance of defects of smaller size is treated as a prefracture stage. The characteristic linear size d, as indicated above, can be determined from comparison of data on quasistatic rupture of defect-free specimens and specimens with stress concentrators (cracks): $d \sim K_{Ic}^2/\sigma_c^2$. In so doing, it is important to bear in mind that correlation is incorrect if between the strength and crack resistance taken from tests on knowingly different scales. To a given characteristic linear size of fracture d there corresponds a certain critical stress σ_c (static ultimate strength) and a characteristic time of preparatory processes τ (incubation time). These parameters (σ_c, K_{Ic}, τ) are strength characteristics on a given scale.

Because fracture on a given scale is understood as the formation of a defect of characteristic linear size d, tests of specimens of smaller size are found incorrect. The parameters determined from data of these tests will correspond to prefracture and will differ greatly. It should be also taken into account that fracture can proceed with the participation of elastic energy accumulated in a region of sizes no larger than

$$D=\tau c,$$

where c is the elastic wave velocity. Therefore, it is appropriate to determine the strength characteristics of a given scale on specimens whose sizes satisfy the inequalities

$$d\leq L\leq D.$$

Thus, the scale of fracture is determined not by one but by two linear sizes — upper and lower. For correct estimation of strength characteristics, tests results on the same scale should be compared. Determination of parameters from comparison of test results related to different scales is incorrect. This fact is important and is to be taken into account in planning tests of materials.

Clearly, there is a scale hierarchy of fracture. Fracture on a larger scale is preceded by fracture on a smaller scale (prefracture stage); the formation of a main crack is preceded by the formation, growth and merging of microcracks. We assume that the upper bound of a scale corresponds to

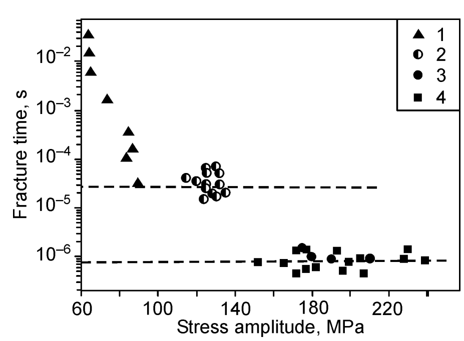

Fig. 2. Two dynamic branches of cleavage strength for polymethylmethacrylat: quasistatic tests (*1*), cleavage failure of rods (separation into parts) [8] (*2*), multisite cleavage of plates [8] (*3*), data of [9] (*4*)

the lower bound of the next scale, i.e., the scale hierarchy can be represented in the form:

$$\ldots < d_{i-1} < D_{i-1} = d_i < D_i = d_{i+1} < D_{i+1} < \ldots .$$

An important consequence of this assumption is that the relationship between different scales is through the incubation time. Moreover, the incubation time can be determined from comparison of data of quasistatic tests corresponding to different scales:

$$\tau_i = \frac{D_i}{c} = \frac{d_{i+1}}{c}.$$

Thus, data of quasistatic tests performed on different scales allow prediction of results of dynamic tests and, vise versa, data of dynamic tests allows prediction of strength characteristics on the next scale.

The possibility to establish the relationship between characteristics of different scales is of fundamental importance for engineering practice. The parameters of a material determined on laboratory specimens of standard sizes can be found unsuitable for adequate analysis of characteristics of large-size constructions, which is now possible to consider as a commonly accepted fact.

4. Nonlocal variant of the structural-temporal criterion

In some cases, practical realization requires generalization of the structural-temporal approach for nonlinear fracture mechanics. The generalization can be of significance for developed plastic deformation that precedes fracture (e.g., in very ductile steels) or where the process covers large zones (e.g., in quasibrittle materials like concrete and rocks with a clearly defined heterogeneous structure). The large zones of the process which are observed at laboratory levels are thus stabilized for specimens of large size to serve as small linear sizes in fracture on large scales with attendant catastrophic brittle fracture like, e.g., in fast crack propagation in main pipelines or in fracture of bulky concrete plates [14, 15]. For analysis of fracture in similar processes, it is efficient to use a "nonlocal" variant of the structural-temporal approach. In this case, the corresponding incubation time criterion is written in the form:

$$\int_{t-\tau}^{t} \int_{x-l_p}^{x} \sigma\,(x', t')\, f(x')\, \mathrm{d}x'\mathrm{d}t' \geq G_l\, \tau, \qquad (8)$$

where $f(x) = -\mathrm{d}w/\mathrm{d}x$ is the weight function of an opening model (with an opening function $w = w(x)$) describing the energy dissipation in a process zone; l_p is the size of the process zone; G_l is the specific fracture energy. The authors of [14, 15] propose various models of the process zone which are in fact reduced to different variants of the weight function $f(x)$. In the simplest particular case of the linear opening function, $f(x) = 1$ and $G_l = \sigma_\mathrm{c} l_\mathrm{p}$, where σ_c is the ultimate strength (quasistatic strength) and τ is the incubation time found from laboratory tests of defect-free specimens for static and dynamic strength. Criterion (7), being dynamic generalization of the classical criterion of linearly elastic fracture mechanics, is derived from (8) in the limiting particular case of linear opening and small process zone.

Let us consider the case of linear opening convenient for practical qualitative estimation. We assume that in tests of specimens sufficiently large for a given scale, the size of the process zone tends to the upper limit [15]:

$$l_\mathrm{p} \to l_{\mathrm{p}\infty} = D = c\tau \;\text{ and }\; G_l \to G_D = \sigma_\mathrm{c} D.$$

Then, in the simplest case (a linear process zone), the prediction of fracture in rather bulky structures made of nonlinear (ductile) materials or brittle materials with large process zones can relay on the limiting condition:

$$\int_{t-\tau}^{t} \int_{x-D}^{x} \sigma(x', t')\, \mathrm{d}x'\mathrm{d}t' \geq G_D \tau. \qquad (9)$$

5. Examples of experimental data analysis

5.1. Dynamic strength of polymethylmethacrylat

The data on the time dependence of strength for polymethylmethacrylat presented in Fig. 2 shows that two dynamic branches correspond to different incubation times — $\tau_1 \approx 0.8$ μs and $\tau_2 \approx 30$ μs. The first value, as shown in [16], fits the characteristic relaxation time due to microcracking of the material. The dynamic branch with the second incubation time was reproduced from cleavage tests of polymethylmethacrylat rods under magnetic pulse loading. It should be noted that the second incubation time is almost coincident with the value (32 μs) found in tests of cracked specimens [17].

5.2. Simulation of dynamic cracking in main pipelines

The diameter of pipelines for which calculation was taken is 1.22 m; in the unrolled state, the pipes are steel sheets of thickness ~8 m, which is surely many times larger than the dimensions of laboratory specimens. A crack in pipelines can travel tens of meters, making full-scale tests extremely expensive. The laboratory specimens to be tested were made of three types of steel: X80, X90 and X100. The static ultimate strength was 625, 711 and 748 MPa, respectively. The incubation time for all test materials was 15 μs, Young's modulus and Poisson's ratio were respectively $E = 2 \cdot 10^{11}$ Pa and $\nu = 0.3$.

The propagation of a crack in a pipeline causes a pressure drop in the pipe and this eventually leads to crack arrest. Despite the sufficient toughness of the pipe material, fracture of the bulky structure proceeds due to accumulated strain energy, i.e., by a quasibrittle scenario. To predict the distance travelled by a crack before its arrest, we are to know the critical stress intensity factor or the characteristic linear size of the fracture zone on a given scale. This parameter was determined by the procedure described above. The time dependence of the crack length was calculated by the finite element method (Fig. 3).

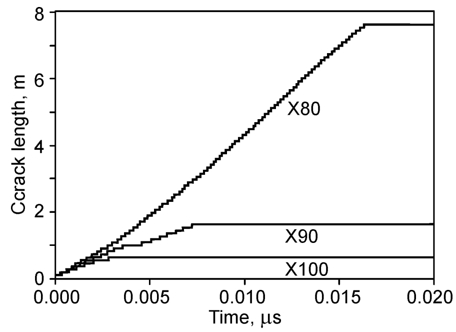

Fig. 3. Crack length versus time for different steel types (calculation by the finite element method)

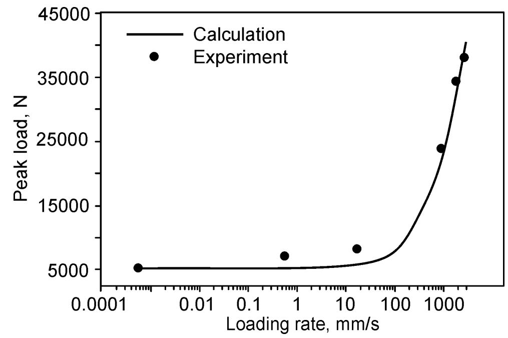

Fig. 4. Peak load versus the loading rate: circles — experimental data of [20], a line — calculation based on the incubation time criterion

The obtained calculation results demonstrated a good agreement with the data of full-scale experiments [18, 19]. Notice that direct use of the crack resistance determined on laboratory specimens would for sure be absolutely inadequate to the simulated process.

5.3. Dynamic strength of concrete

The scale effect is particularly pronounced in heterogeneous materials such as concrete and rocks. In this case, one should apply nonlocal criterion (9).

The authors of [20] report on the data of fracture tests of high-strength concrete specimens at a varying loading rate. The tests were performed by the pattern of three-point bending. The specimen cross-section was 100×100 mm; the spacing between beam seats was 300 mm; and the initial crack length in the beam was 50 mm. The specimens were loaded by a force applied to an impactor; in so doing, a specified displacement velocity of the point of contact of the impactor and specimen was ensured. The highest force arising under loading was determined for six different displacement velocities of the point of contact ($5.5\cdot10^{-7}$, $5.5\cdot10^{-4}$, $1.74\cdot10^{-3}$, $8.81\cdot10^{-1}$, 1.76 and 26 m/s).

Analysis of the experimental data was made using the following parameters of the material (high-strength concrete): the specific formation energy of a new surface 147.5 N/m and the rupture strength 6.8 MPa ($P = 40$ kN/mm).

For all six loading rates realized in the experiments, the measured dependence of displacement of the point of contact on the loading force is close to linear to the point of fracture of the specimen.

Using the incubation time criterion, we can calculate the point of fracture and the peak load for each loading rate. It was found that the incubation time for the test material is 1.3 ms.

Figure 4 shows calculated and experimental dependences of the peak load on the loading rate. It is seen that they are in a good agreement.

Thus, the incubation time for fracture in high-strength concrete under the experimental conditions (i.e., on the given scale) was found. Following the approach reported above, one can determine the upper scale D which, according to the assumptions made, is the characteristic defect size d for the next scale: $D = C\tau$.

Because the longitudinal wave velocity for high-strength concrete is $c = 4.3\cdot10^3$ m/s, the lower characteristic size of the next scale is found equal to 5.85 m. This value agrees well with numerous experiments, e.g., [15], according to which the fracture conditions for concrete constructions of dimensions beginning with 6 m can be estimated with the use of linear fracture mechanics; that is the strength of concrete constructions with a characteristic size larger than 6 m can be analyzed using the model of a homogeneous isotropic defect-free material. This fact supports the applicability of the developed approach to fracture prediction on the next scale.

6. Conclusion

Although the ideas and preliminary results reported in the work require further experimental and theoretical substantiation, we can point to a series of issues that are fundamentally important for research in the multiscale nature of fracture:

– Correct determination of fracture scales is possible only if based on consideration of dynamic peculiarities of the process and spatial-temporal approach.

– A fracture scale is determined by the conditions of experimental measurements of parameters and is characterized by two linear sizes — upper and lower.

– In tests of materials, it is required to trace implicit (incorrect) transitions from one scale to another which are a consequence of changes in experimental conditions and measuring techniques.

– There is a fundamental possibility to predict the fracture dynamics on one scale from parameters determined on another scale. The development of industrial methods of this prediction is a major problem for engineering practice.

References

[1] V.E. Panin, Yu.V. Grinyaev, and V.I. Danilov, Structural Levels of Plastic Deformation and Failure, Nauka, Novosibirsk, 1990 (*in Russian*).

[2] V.E. Panin, V.A. Likhachev, and Yu.V. Grinyaev, Structural Levels of Deformation in Solids, Nauka, Novosibirsk, 1985 (*in Russian*).
[3] J. Nicolis, Dynamics of Hierarchical Systems: An Evolutionary Approach, Springer, Berlin, 1986.
[4] V.N. Kukudzhanov, Numerical Modelling of Deformation, Damage and Fracture of Materials and Constructions, MFTI, Moscow, 2008 (*in Russian*).
[5] Y.V. Petrov, A.A. Gruzdkov, and N.F. Morozov, The principle of equal powers for multilevel fracture in continua, Dokl. Phys., 50, No. 9 (2005) 448.
[6] V.V. Novozhilov, On the necessary and sufficient criterion of brittle fracture, Prikl. Mekh. Tekhn. Fiz., 33, No. 2 (1969) 212 (*in Russian*).
[7] N.A. Zlatin, S.M. Mochalov, G.S. Pugachev, and A.M. Bragov, Temporal mechanisms of metal fracture under severe loads, FTT, 16, No. 6 (1974) 1752 (*in Russian*).
[8] E.N. Bellendir, Experimental Study of Brittle Fracture of Solids in Tensile Stress Waves. Cand. Degree Thesis (Phys&Math), Ioffe PTI AS USSR, Leningrad, 1990 (*in Russian*).
[9] E.P. Evseenko, E.L. Zilberbrand, N.A. Zlatin, and G.S. Pugachev, Dynamical branch of time dependence of polymethylmethacrylate strength, Pisma ZhTF, 3, No. 14 (1977) 684 (*in Russian*).
[10] V.S. Nikiforovskii and E.I. Shemyakin, Dynamic Fracture of Solids, Nauka, Novisibirsk, 1979 (*in Russian*).
[11] Y.V. Petrov, On “quantum” nature of dynamic fracture of brittle media, Dokl. AN SSSR, 321, No. 1 (1991) 66 (*in Russian*).
[12] Y. Petrov and N. Morozov, On the modeling of fracture of brittle solids, ASME J. Appl. Mech., 61 (1994) 710.
[13] V.A. Bratov, N.F. Morozov, and Y.V. Petrov, Dynamic Strength of Continuum, St-Petersburg University Press, St-Petersburg, 2009.
[14] B.L. Karihaloo, Size effect in shallow and deep notched quasi-brittle structures, Int. J. Fract., 95 (1999) 379.
[15] B.L. Karihaloo, H.M. Abdalla, and Q.Z. Xiao, Deterministic size effect in the strength of cracked concrete structures, Cement. Concrete Res., 36 (2006) 171.
[16] P.A. Glebovskii, Microscopic physical peculiarities of fracture at pulse loading, Vestn. Mol. Uch., Prikl. Mat. Mech., No. 2 (2003) 49 (*in Russian*).
[17] S.I. Krivosheev, N.F. Morozov, Y.V. Petrov, and G.A. Shneerson, Fracture initiation in solids under severe pulse loading, Izv. RAN, Mekh. Tv. Tela, 65, No. 5 (1999) 165 (*in Russian*).
[18] A.I. Abakumov, Numerical Simulation of Crack Propagation in Trunk Gas Pipelines, in *Proc. 16th Int. Conf. Pipes 2008, 15–17 September 2008, Chelyabinsk, Russia*.
[19] S. Igi and T. Akiyama, Dynamic Ductile Fracture Analysis for Large Diameter X80 Pipelines, in *Proc. 16th Int. Conf. Pipes 2008, 15–17 September 2008, Chelyabinsk, Russia*.
[20] G. Ruiz, X. Zhang, J. del Viso, R. Yu, and J. Carmona, Influence of the loading rate on the measurement of the fracture, Anales Mec. Fract., 25 (2008) 793.