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MECHANICS

Simulation of Dynamic Crack Propagation under Quasi-Static Loading

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It is known that, within the framework of linear fracture mechanics (LFM) under quasi-static loading, the mechanical-stress field in the vicinity of the tip of a symmetrically loaded crack represented by a mathematical linear cut is determined by stress-intensity factor (SIF) K_I for the first loading mode, the corresponding critical value of which is found experimentally. The criterion of the critical intensity factor widely used in engineering practice was extended in many works also to the dynamic-fracture case [1, 2]:

$$K_I(t, P(t), \Omega(t), \dot{L}(t)) \le K_{Id}(\dot{K}_I(t), T, \ldots). \quad (1)$$

Under this limiting condition, $P(t)$ is the general dynamic loading, $\Omega(t)$ is the current sample configuration including the time-varying crack size $L(t)$, and $\dot{L}(t) = \frac{dL}{dt}$ is the current crack-propagation velocity. It is assumed that the equality sign is implemented in Eq. (1) during the propagation of the crack from the moment of starting to the moment of stopping. On the right in Eq. (1), there is a function called the fracture dynamic viscosity K_{Id}, which is in most cases considered as the material function of the local (in the vicinity of the crack tip) loading rate $\dot{K}_I(t) = \frac{dK_I}{dt}$, the temperatures T, and other characteristics of the experiment. When calculating the fracture process, the right-hand side in Eq. (1) is assumed a priori known and is determined from experiments for this material. Now this approach is reasonably widely spread in the dynamic-fracture calculations. Nevertheless, many experimental results obtained, for example, by the authors of [3–5], impugn the analysis based on using criterion (1) and, in particular, the presence of a direct dependence between current critical intensity factor K_I derived from the LFM principles and crack propagation velocity. In [3–5], it is established that in the case when the samples are exposed to high-rate shock loading initiating fast crack growth, the crack velocity can remain practically constant even at very significant variation in the current intensity-factor values. The authors of the papers [3–5] assumed that the energy flux determined by the current stress-intensity factor is unambiguously independent of the macroscopic crack-propagation velocity, although it significantly affects the structure of the newly formed surface. Thus, the conclusions made in [3–5] reject the method based on using the LFM and limiting condition (1) accepted in many investigations.

It turned out that the effects of the behavior of cracks for high-speed shock loadings observed in [3–5] are predicted well and simulated on the basis of the structural–time approach based on the concept of the fracture incubation time [5–7].

On the other hand, a quite steady dependence of the crack-propagation velocity $\dot{L}(t)$ on its length $L(t)$ (which is treated also as the dependence on $K_I \sim \sqrt{L}$) is sometimes observed in many experiments. For example, it can be observed in experiments [9, 10] in which the samples in the form of flat plates with initial macrocracks were exposed to a slow quasi-static stretching, which at a certain moment resulted in the start, then the acceleration, and the subsequent fast dynamic propagation of a crack through the entire plate. The behavior observed in [9, 10] does not principally contradict the scheme in the basis of criterion (1); however, it requires a very technologically complex and expensive determination of the functional in the right-hand side of this limiting equation. Besides this, the utilized rate dependences of fracture toughness are highly unstable—it is observed in many experimental works. This problem is not resolved within this scheme.

When comparing the experiments carried out under various conditions for the same material, it is

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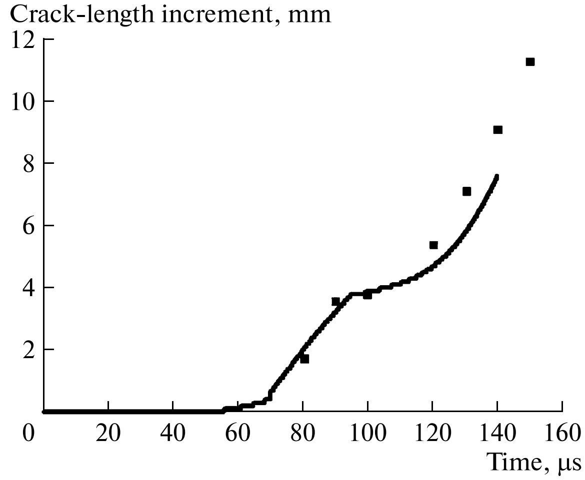

Fig. 1. Results of simulation of tests with dynamic loading of samples; the curves are the simulation [11], and the points are experimental data [3].

possible to conclude that the critical stress-intensity factor, nevertheless, cannot be considered as a material function invariant with respect to the history and conditions of loading, which completely determines dynamic crack propagation even with rate dependences taken into account explicitly (as, for example, in (1)).

In [6–8], it was shown that it is necessary to take into account the incubation processes accompanying material macrofracture to explain crack propagation under the action of high-rate shock loadings. On this basis, we successfully numerically simulated in [11] the experiments [3, 4] describing the onset of crack propagations, its motion, and crack arrest. The results of the simulation are shown in Fig. 1.

We show that the results of both types of experiments mentioned can be explained successfully using the structural–time approach. We perform numerical simulation of experiments from [9, 10] implementing the method based on introducing the incubation time fracture criterion [11] into the finite-element scheme. The corresponding theory of dynamic fracture is formulated in [6–8]. According to it, the critical condition of the material failure at a point x and at the moment of time t can be written as follows:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x', t')dx'dt' \le \sigma_c, \tag{2}$$

where τ is the incubation time characteristic for the fracture process at this scale level, which is a constant of the material independent of the type of loading and the sample geometry; and d is the characteristic size of the zone in which the sample fracture takes place. It should be noted that d is also the material constant for this material and for a certain scale level. Here, $\sigma(x, t)$ is the stress at the point under study at the moment of time t, σ_c is the critical stress obtained in static experiments with the samples, the sizes of which agree with the boundaries of this scale level [12]. We note that d is unnecessarily related unambiguously to certain internal geometrical properties of the material, for example, with the crystal-lattice-cell size. We accepted the interpretation of d as the scale fit parameter for the strength characteristics determining the lower boundary of the scale level on which the fracture process is investigated [12, 13]:

$$d = \frac{2}{\pi}\frac{K_{Ic}^2}{\sigma_c^2}, \tag{3}$$

where K_{Ic} and σ_c are assumed as measured at the same scale level. Thus, Eq. (3) enables us to calculate the characteristic fracture-zone size d determining the lower scale boundary or the smallest element of fracture for this scale level [12].

Following the scheme [9, 10], we consider the initial-boundary-value problem for a PMMA plate with the following size ranges: 10–20 cm in width, 14–25 cm in height, and 1.6–3.2 mm thick (for the numerical simulation, we selected samples with the largest sizes in width and height and 3 mm thick). On the plate, an initial crack of 4–6 mm in length is formed. Further, the upper and lower edges of the sample are fixed in holders of the extension machine, which start to move with a reasonably low speed for neglecting the wave effects. The authors of [9, 10] had the possibility to trace the crack-tip position and to measure its propagation velocity. In addition, the stress necessary for the crack start, which was used in the simulation, was fixed.

Further, we assumed linear-elastic behavior of the flat area simulating the plate at all moments of time. The displacement and stress fields are determined by the dynamic equations of the linear theory of elasticity

$$\begin{gathered}\rho\frac{\partial^2 U_i}{\partial t^2} = (\lambda+\mu)\nabla_i(\nabla\cdot\bar{U}) + \mu\Delta U_i,\\ \sigma_{i,j} = \delta_{i,j}\lambda\nabla\cdot\bar{U} + \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\end{gathered} \tag{4}$$

with the corresponding boundary and initial conditions

$$\begin{gathered}\bar{U}(X, t=0) = \frac{\partial\bar{U}}{\partial t}(X, t=0) = 0,\\ \sigma_{i,j}(X, t=0) = \frac{\partial\sigma_{i,j}}{\partial t}(X, t=0) = 0,\\ U_y(X\in\Gamma_1, t) = vt,\end{gathered} \tag{5}$$

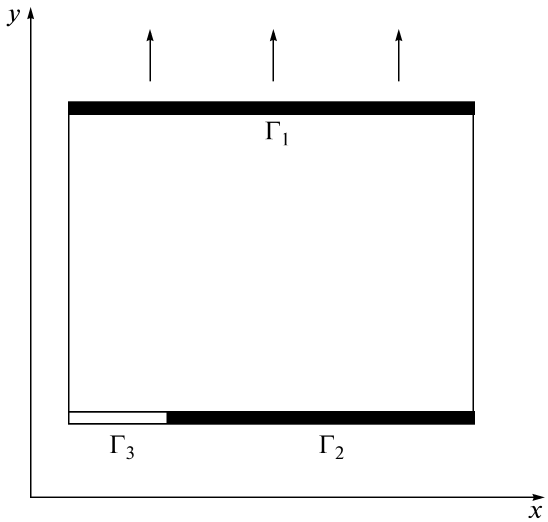

Fig. 2. Scheme of J. Fineberg simulation experiments [9].

$U_y(X \in \Gamma_2, t) = 0$ is the symmetry condition,

$$\sigma_y(X \in \Gamma_3, t) = \sigma_{xy}(X \in \Gamma_2 \cup \Gamma_3, t) = 0.$$

Here, $X = (x_1, x_2) = (x, y)$ and $\overline{U} = (U_1, U_2) = (U_x, U_y)$; v is the velocity of motion of holders of the extending machine; LAMBDA and MU are Lame coefficients. Due to the symmetry of the problem, we simulated only half of the plate. In this case, the symmetry line coincides with the crack-propagation trajectory (see Fig. 2). As the initial moment of time, we accepted the moment of start of the motion of holders.

The simulation is carried out in the finite-element package ANSYS with using the individual program modulus providing fracture criterion (2) check and implementing the crack propagation.

The finite element size was chosen due to used fractute criterion—the side of square elements along crack propagation path equals 0.2 mm. This value coincides with the size of the fracture structural cell for experiment scale level (3).

The values of K_{Ic} and σ_c were obtained for the quasi-static tests of samples comparable with dimensions of plates in the simulated experiments. Thus, their use is correct for the calculation of d for the simulation. It should be noted that, at such a choice of the element size, the smallest crack length increment equals d, which correctly agrees with the interpretation of this parameter as the characteristic fracture-zone size.

The crack propagation is implemented due to removal of fastening in a lattice site in which criterion (2) was fulfilled.

In the table, we listed the material parameters that were used in the simulation. Figure 2 shows the results

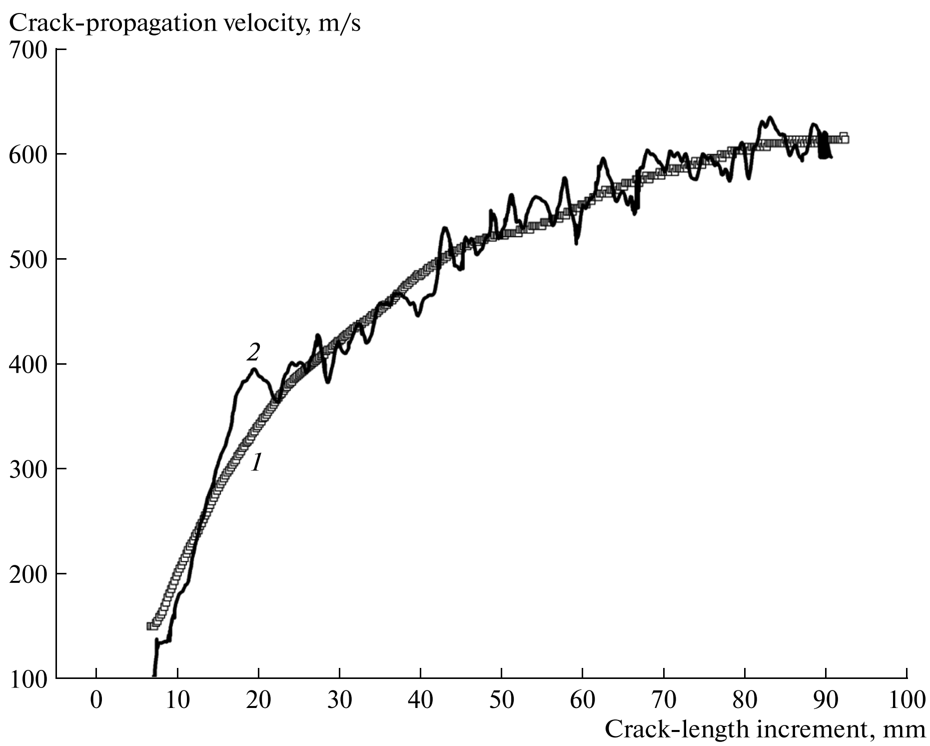

Fig. 3. Results of simulation of tests with quasi-static loading. Dependence of crack velocity on crack length; *1* is the simulation, and *2* is the experiment [9].

Table

Young's modulus E	3.5 GPa
Poisson's ratio ν	0.32
Density ρ	1200 kg/cm^3
Critical stress σ_c	60 MPa
Critical SIF	1.1 MPa $\sqrt{m}$
d	0.2 mm
τ	1.5 μs

of simulation (the light strip) and the experimental data (the dark strip). The crack velocity in the steady mode of motion quite precisely coincides with the experimental values. At the start, the crack moves with very high acceleration, then, the crack velocity stabilizes.

Thus, it is shown that the use of the structural–time approach and the fracture incubation-time criterion enables us to predict successfully the results of experiments both on quasi-static [9, 10] and on dynamic [3–5] loading of samples with cracks.

The proposed approach makes it possible to reject the conventional method based on introducing the dynamic fracture toughness K_{Id}, which actually is not a material property and cannot be efficient for describing the dynamic fracture due to its strong dependence on the loading history [6–8]. From [3–5, 11] and the calculations carried out, it also becomes clear that the dependence of the crack velocity on the stress-intensity factor cannot be considered as a material law because the properties of this dependence are determined by the sample configuration, the history, and the loading method.

The correct prediction of the dynamic fracture is possible on the basis of the structural–time criterion taking into account the incubation processes accompanying the dynamic material failure and containing the new physical parameter—the incubation time, which is the material constant within the limits of the chosen scale level. The incubation time can be found in independent experiments and, then, used for predicting fracture in a wide range of loading rates—from slow quasi-static to the high-rate shock-wave ones. Supplementing it with conventional reference parameters and the simply calculated structural-element size d, it is possible, as shown above, to predict a wide spectrum of the effects accompanying the dynamic growth of cracks. Thus, the developed approach can be considered as a reasonably simple but, at the same time, efficient tool for both theoretical investigations and engineering practice.

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