

# Existence of Optimal Energy Saving Parameters for Different Industrial Processes

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**Abstract:** It is demonstrated that energy input for fracture in many industrial processes can be optimised so that the energy cost of the process is minimised. Using a simple example of central crack it is shown that for a certain shape of the load pulse energy transmitted to the sample in order to initiate the crack has a strongly marked minimum. Received results indicate a possibility to optimise energy consumption of different industrial processes connected with fracture. Possible applications include drilling or rock pounding where energy input often accounts for the largest part of the process cost. Using this approach it will be possible to predict optimal operational parameters for bores, grinding machines, etc. and hence significantly reduce the process cost. In the second part of the paper the behaviour of energy input for initiation of fracture in conditions of contact interaction is studied. It is considered that a spherical particle is impacting the half-space. Stress field created as a result of the interaction can be estimated using the Hertz solution. Threshold particle velocity (and, hence, threshold kinetic energy) corresponding to initiation of rupture in the half-space can be found once the fracture criterion is defined. It will be shown that the value of this energy does significantly depend on load duration and has a marked minimum. Existence of energetically optimal modes of dynamic impact is claimed.

**Key words:** energy saving; optimal operational mode; incubation time; brittle fracture; rocks

**CLC number:** O34

**Document code:** A

**Article ID:** 1001-5132(2012)01-0048-05

The possibility of optimizing the amount of energy required to fracture materials is of great interest in connection with many applications. Energy inputs for fracture induced by short impulse loadings are of the major importance in such areas as percussive, explosive, hydraulic, electro-impulse and other means of mining, drilling, pounding etc. In these cases energy input usually accounts for the largest part of the process cost (e.g. [1]). Taking into consideration the fact that the efficiency of the mentioned processes rarely exceeds a few percent, the importance of energy inputs optimiz-

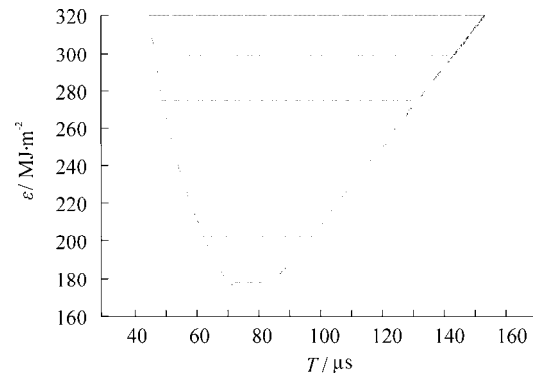
ation becomes evident.

## 1 Estimation of energy required to initiate central crack in a plate

In [2] an attempt to estimate the amount of energy sufficient to initiate mode I loaded central crack in a plate subjected to plane strain deformation was made. The study involves analysis of interaction of the wave approaching from infinity with an existing central crack in a plane. The existing crack is oriented parallel to the

front of the wave. The process was analyzed utilizing the finite element method. The ABAQUS finite element package was used to solve the problem. Computations were performed for granite ( $E = 96.5$  GPa,  $\rho = 2810$  kg·m<sup>-3</sup>,  $\nu = 0.29$ , where  $E$  is the elasticity modulus and  $\nu$  is the Poisson's ratio). The results of the investigation will qualitatively hold for a large variety of quasi-brittle materials.

Fracture criterion fulfilment was checked for different load amplitudes and durations. Dependence of time-to-fracture  $T^*$  on the amplitude of the load applied was investigated. Time-to-fracture is the time from the beginning of interaction between the wave package and the crack to fracture initiation. Incubation time criterion of fracture<sup>[3-4]</sup> was adopted. A similar approach to be used in the case of short cracks is given in [5]. Using the incubation time criterion, the dependence of time-to-fracture on the amplitude of the load pulse applied was studied. Values of  $K_{IC} = 2.4$  MPa $\sqrt{m}$  and  $\tau = 72$   $\mu$ s typical for the granite were used. Integration of the temporary dependence of the stress intensity factor was done numerically. Energy transmitted to the sample by a virtual loading device in the process of impact can be evaluated analytically. A specific (per unit of length) energy transmitted to the stripe can be calculated using the solution for the uniformly distributed load acting on a half plane. This problem can be easily solved utilizing the D' Alembert method. Fig. 1 is giving a limiting curve for a set of energies that, being transmitted to the sample, cause the crack propagation. The minimum energy able to increment the crack length ( $172e6$  J) is reached at load pulses with duration of  $78$   $\mu$ s. As it is evident from Fig. 1, the minimum energy, required to propagate the crack by impacts with durations differing greatly from the optimal one, significantly exceeds the minimum possible value. Thus the minimum energy, initiating the crack for the load with duration of  $92$   $\mu$ s (at this impact duration crack propagation is possible with the impact of threshold amplitude), will exceed the minimum energy possible by 10%, and at duration of  $40$   $\mu$ s it will be more than two times bigger.



**Fig. 1 Filled area corresponds to a set of possible pulses leading to crack initiation (at  $T = 72$   $\mu$ s energy needed to advance the crack is minimized)**

With the majority of non-explosive methods used to fracture materials (drilling, grinding etc.), it is possible to control amplitude and frequency of impacts from the side of a rupture machine. The performed modeling shows that at a certain load duration (at impact fracture of large volumes of material, impulse duration is connected to the frequency of the machine impacts) energy input for crack propagation has a marked minimum. Analogously to Fig. 1, it is possible to plot the limiting curve for the set of energy values leading to propagation of a crack in the sample at different load amplitudes. Thus, it is possible to establish ranges of amplitudes and frequencies of load, at which energy costs for fracture of the material are minimized. These ranges are dependent on parameters of fractured material, predominant length of existing material cracks and the way the load is applied.

Dependence of the optimal load parameters on the crack length was also studied. It was found that the duration of the load, that minimizes energy, and momentum inputs are linearly or quasi-linearly dependent on the existing crack length. With the disappearing crack length, the duration of the load minimizing momentum needed for crack propagation approaches zero. At the same time, the duration optimal for the energy inputs tends to the micro-structural time of the fracture process. The maximum possible time-to-fracture also tends to the micro-structural time of fracture.

## 2 Minimization of fracture energy in the case of contact interaction

While analyzing dynamic strength of materials, one is facing a contradiction between available experimental results and classical quasi-static approaches in fracture. Numerous experiments demonstrate that under high-rate dynamic loads, materials are able to endure loads significantly exceeding fracture loads in static (quasi-static) conditions. At the same time, in some of the experiments fracture in dynamic conditions is initiated at a moment when local stresses at a rupture point are significantly less as compared to stresses leading to fracture initiation in static (quasi-static) conditions. These obvious contradictions led to attempts to “correct” and “generalize” classical fracture criteria in order to make it applicable in the case of high-rate loads. This led to the appearance of a concept of “dynamic strength”, depending not only on the material properties, but also on the loading rate and even the time-shape of the load pulse<sup>[6-7]</sup>. Practical utilization of this approach is rather complicated and often impossible, as there is no possibility of evaluating dynamic strength for all variety of loading rates and load shapes.

Most researchers dealing with problems of dynamic fracture are using fracture criteria based on extrapolation of quasi-static fracture criteria to dynamic conditions. Though they normally account for inertia and temporal characteristics of the load applied, temporal characteristics of the fracture process are usually not taken into consideration. Utilizing this kind of approach, it is impossible to predict a critical situation, leading to fracture, applicable to both dynamic (high-rate loads) and quasi-static cases. In this section, an incubation time fracture criterion is used in order to predict fracture in the case of contact interactions. Employing this approach one does not need to worry about the time scale of the problem: the criterion gives correct predictions in a wide range of loading rates from static problems to the extremely dynamic ones. For the present analysis, we need to consider a wide range

of loading rates and load durations. In this regard the incubation time fracture criterion provides a unique possibility to achieve correct estimations of conditions leading to fracture for the complex problem of rocks spudding. Suppose that the shape of a loading pulse can be approximated by a smooth function:

$$\omega(t) = \begin{cases} \exp(1/(1-(2t/t_0-1)^{-2})), \\ |t-t_0/2| \leq t_0/2, \\ 0, |t-t_0/2| > t_0/2, \end{cases}$$

where  $t_0$  is for the load duration. Then the load is given by

$$\sigma(t) = \sigma_{\max} \omega(t), \quad (1)$$

where  $\sigma_{\max}$  is the load amplitude. Substituting equation (1) into incubation time fracture criterion, one can obtain the critical amplitude  $\sigma_{\max}$  leading to fracture and corresponding to execution of the incubation time criterion:

$$\sigma^* = \sigma_c \tau / \max_{t \in [0, t_0]} \int_{t-\tau}^t \omega(s) ds.$$

As an option for the way the energy is delivered to the fracture zone, consider a problem of impact interaction. In [8], Petrov et al analyzed a problem for a spherical particle having radius  $R$  and velocity  $V$  impacting an elastic half-space using the classical Hertz contact scheme. The maximal stresses appearing in the half-space and the duration of interaction between the particle and the half-space were calculated. According to the Hertz hypothesis, the contact force  $P$  arising between the particle and the half space can be presented as

$$P(t) = kh^{3/2}, \\ k = 4/3\sqrt{R} \frac{E}{(1-\nu^2)}, \quad (2)$$

where  $h$  is a particle penetration and  $\nu$  is the Poisson's ratio of the elastic media. The maximal penetration  $h_0$  can be found as

$$h_0 = (5mV^2 / (4k))^{2/5}, \quad (3)$$

where  $m$  is the mass of a particle. The impact duration can be presented as

$$t_0 = \frac{2h_0}{V} \int_0^1 \frac{d\gamma}{1-\gamma^{5/2}} = 2.94 \frac{h_0}{V}, \quad (4)$$

the dependence of time on penetration  $h(t)$  can be approximated by

$$h(t) = h_0 \sin(\pi t / t_0), \quad (5)$$

time-dependent maximum tensile stress generated in the impacted media can be estimated by

$$\sigma(V, R, t) = \frac{1-2\nu}{2} \frac{P(t)}{\pi a^2(t)}, \quad (6)$$

where the radius of the contact area  $a(t)$  is given by

$$a(t) = (3P(t)(1-\nu^2) \frac{R}{4E})^{1/3}. \quad (7)$$

Knowing the duration and amplitude of the applied load, the mass and velocity of the impacting particle can be found from equations (2) – (7):

$$R = \frac{t_0}{2.94} \left( \frac{6}{5} \frac{\sigma_{\max}}{\rho(1-2\nu)} \right)^{1/2},$$

$$V = \left( \frac{5}{4} \frac{\rho \pi (1-\nu^2)}{E} \right)^2 \left( \frac{6}{5} \frac{\sigma_{\max}}{\rho(1-2\nu)} \right)^{5/2},$$

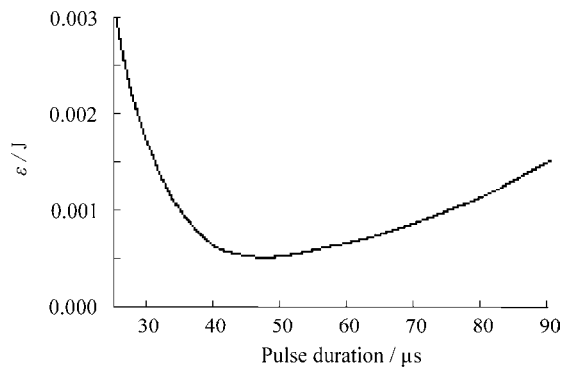
where  $\rho$  is a parameter of load intensity, having a dimension of mass density, and  $\sigma_{\max}$  is the maximum stress (i.e. load amplitude). Evaluating the initial kinetic energy of the spherical particle, one can estimate the energy required in order to create fracture in the impacted media

$$\varepsilon = \alpha \frac{t_0^3 \sigma_{\max}^{13/2}}{\rho^{3/2} E^4},$$

where

$$\alpha = \frac{2}{3} \frac{\pi^5}{(2.94)^3} \left( \frac{5(1-\nu^2)}{4} \right)^4 \left( \frac{6}{5(1-2\nu)} \right)^{13/2}$$

is a dimensionless coefficient. This energy, corresponding to the value  $\rho = 2400 \text{ kg}\cdot\text{m}^{-3}$ , is plotted



**Fig. 2 Energy necessary to fracture versus load pulse duration for gabbro-diabase**

versus impact duration in Fig. 2. The properties of the material are taken to be equal to the properties of gabbro-diabase<sup>[9]</sup> ( $E = 6.2 \times 10^9 \text{ N}\cdot\text{m}^2$ ,  $\sigma_c = 44.04 \times 10^6 \text{ N}\cdot\text{m}^2$ ,  $\nu = 0.26$  and  $\tau = 440 \text{ μs}$ ).

### 3 Conclusions

The results received indicate a possibility of optimising the energy consumption of different fracture-connected industrial processes (e.g., pounding, drilling). It is shown that the energy cost of crack propagation strongly depends on the amplitude and duration of the load applied. For example, in the studied problem when the duration of the load differs from the optimal one by 10%, the energy cost of initiating the crack is exceeding the minimum value by more than 10%. The obtained dependencies of the optimal characteristics of a load pulse on the existing crack length can help in predicting energy-saving parameters for the fracture processes by investigating the predominant crack size in a fractured material. Knowing the fracture incubation time for the particular material we can select the most energetically favorable mode of treatment. In particular, adjusting the duration of impacts, we can optimize the operation of rupture devices of the impact type. Similarly, it is possible to choose the vibration modes for decreasing the energy losses during processing of various materials. Thus, it was demonstrated that the incubation time approach is providing a possibility to predict the strength of rocks in a wide range of loading rates as well as to optimize the energy input needed to create rupture of rock media.

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## 各种生产过程最优节能参数的存在性

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摘要: 研究表明, 在诸多生产过程中可以优化输入的断裂能量, 使加工能耗最小. 以一个简单的中心断裂为例, 当加载脉冲具有某种特定形态时, 施加到样品上引发破裂的能量可以取一个显著的最小值. 研究结果显示, 优化各种与断裂有关的生产过程, 使得能量消耗最小是可能的. 这种优化的可能应用包括钻孔或岩石粉碎过程, 此时能耗成本通常占生产成本的最大部分. 上述方法可以预测钻孔或研磨机等的最优参数, 从而大大降低生产成本. 文章第二部分研究了接触作用条件下引发断裂的能量输入行为. 考虑一个球型粒子冲击一个半空间靶的情形, 接触导致的应力场可用赫兹解来估计. 定义了断裂判据之后, 即可确定引发断裂的球粒临界速度及相应的临界动能. 该动能的大小依赖于加载的持续时间, 并十分明显地存在一个最小值. 据此可断定, 存在一个动态冲击的能量最优模式.

关键词: 节能; 最优工作模式; 孕育时间; 脆性断裂; 岩石

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