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EVALUATION OF FRACTURE INCUBATION TIME FROM QUASISTATIC TENSILE STRENGTH EXPERIMENT

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Abstract. Incubation time being the main characteristic parameter for dynamic fracture process is experimentally measured for PMMA utilizing optical methods. The specimen is quasistatically loaded in standard tensile testing machine until brittle fracture occurs when the sample is split into two parts. Normally this splitting of brittle materials is accompanied by the impact unloading of the sample. In considered tests tensioned samples were dynamically unloaded by stress drop wave, generated by fracture process and registered by photoelasticity technique at a certain distance from breaking line. The same experiment is simulated using ANSYS FEM software package and the incubation time is evaluated numerically. The simulation results are in a good coincidence with experimental measurements, proving the applicability of the proposed simple method for brittle fracture incubation time measurements.

1. Introduction

It is known and nowadays generally accepted (see ex. [1, 2]) that dynamic fracture caused by intensive transient loads (for example explosive of intense impact load) cannot be predicted on the basis of classical fracture mechanics. Numerous experimental results [3, 4] reveal contradictions with classic approaches (i.e. critical stress or critical stress intensity factor concepts) that can only be explained by inapplicability of static approaches in dynamic problems. In other words, transient processes including for example small-scale damage, precessing macroscopic fracture or medium inertia should be taken into account.

Spatial dimension being introduced into fracture criteria is providing a possibility for correct prediction of quasistatic fracture for problems with non square root stress singularities. This type of criteria was originally proposed by Neuber [5] and Novozhilov [6]:

$$\frac{1}{d}\int_{x^*-d}^{x^*}\sigma(x)dx \le \sigma_C. \qquad (1)$$

Here σ_C is the material ultimate stress, $\sigma(x)$ stands for the stress in point x and x^* is the location of fracture. Size d can be received as $d = 2K_{IC}^2/\pi\sigma_C^2$, where K_{IC} is the critical stress intensity factor, from the requirement of coincidence of (1) with Irwin-Griffith critical stress intensity factor fracture criterion in the case of square-root singularity. This size may be treated as a characteristic size of a fracture process zone being a scale level identifier [7]. This is a minimal size for a damaged medium that can be called “fracture” at a chosen scale level (e.g. minimal length of a microcrack in a problem of crack propagation). Currently the criterion (1) is included as a special case into the incubation time fracture approach [7-10] introducing spatial-temporal discretization of fracture process. This criterion, along with the characteristic length d is introducing the characteristic time τ needed for formation of fracture on a preset scale level. This so called *incubation* time τ is a material propertyand is responsible for transient features of the fracture process. Fracture on each scale level is a result of complicated kinetic processes such as growth and coalescence of microcracks [3, 4] and therefore should never be regarded as an instantaneous event. Incubation time makes it possible to take these microscale processes into consideration and hence provides a possibility to solve nonlinear problem of dynamic fracture staying within the framework of linear formulation.Incubation time fracture criterion, originally proposed to predict crack initiation in dynamic conditions, was formulated in [7-8]. This criterion for fracture at a point x^*, at time t^* , reads as:

$$\frac{1}{\tau}\int_{t^*-\tau}^{t^*}\frac{1}{d}\int_{x^*-d}^{x^*}\sigma(x,t)dxdt \leq \sigma_C. \quad (2)$$

In (2) stress field in the material is time dependent and integration over time indicates that the history of stresses is taken into account or in other words the information about processes preceeding fracture is controlled by a single measurable parameter – τ. t^* is the moment of time when macroscopic fracture (for a given scale level) takes place. For a specified fracture point x^* the value of t^* may be calculated from (2) if the function $\sigma(x,t)$ is known.

As it was shown in multiple publications (ex. [10-14]), criterion (2) can be successfully utilized to predict fracture initiation in brittle solids. For slow loading rates and, hence, times to fracture that are essentially bigger than τ, condition (2) for crack initiation gives the same predictions as Irwin's criterion of the critical stress intensity factor [9]. For high loading rates and times to fracture comparable with τ all the variety of effects experimentally observed in dynamical experiments (ex. [1, 3-4]) can be received using (2) both qualitatively and quantitatively [11]. Application of condition (2) to prediction of real experiments or usage of (2) as a critical fracture condition in finite element numerical analysis gives a possibility for better understanding of fracture dynamics nature (ex. [11, 12]) and even prediction of new effects typical for dynamical processes (ex. [14]).

Therefore, having a reliable and rather simple technique to measure τ for a wide range of materials is of a vital importance both for practical applications and the future dynamic fracture research.

2. Experimental technique to measure brittle fracture incubation

One of the possible ways to determine incubation time experimentally was proposed in work by Krivosheev and Petrov [15-16]. The experiments were carried out on a unique magnetic pulse installation, being able to generate short-time pressure pulses. Single edge notched PMMA specimens were tested via pressure pulse application to initial crack faces. Due to the fact that pressure pulse duration and magnitude could be controlled by researchers, minimal (or threshold) magnitudes for crack initiationwere found. The authors managed to obtain incubation time for PMMA equal to $30\,\mu s$ using formula that involves the threshold amplitude, solution of dynamic initial boundary value problem and some material properties such as wave velocities and critical stress intensity factor K_{IC}.

In general any experimental machine that could provide dynamic loading with known impulse data would suit to measure threshold loading parameters and therefore calculate incubation time of the process. The significant drawback of such a straightforward method is its extreme complexity and high cost. The machines that are able to create short controllable pulses are usually unique and very difficult to use. Such approach is not applicable for routine engineering needs.

In this paper we propose another method for incubation time evaluation which is based on simple quasistatic sample testing and photoelastic properties of PMMA specimens [17].

Suppose we perform a classic tensile test on a standard flat sample. Quasi brittle fracture of the tested material is supposed. At some moment of time$t$ the sample is divided into two parts as the stress in the sample reaches critical value P. Following the classic concepts of linear elasticity fracture event should result in an instantaneous stress drop at a fracture point. This stress drop would generate a step shaped relaxation wave in the sample. The stress at the fracture point may be represented by $\sigma(t) = P - PH(t)$ with $H(t)$ being the Heaviside step function.

However for real processes such a suggestion contradicts the nature. It takes time for the fracture process to develop from micro scale to macro scale,material needs time in order to accelerate and start moving. In other words, failure should not be represented as an instantaneous event, as it is a continuous process in time. According to this natural assumption stress as a function of time in the break point can be presented as: $\sigma(t) = P - Pf(t)$, where $f(t)$ is some function without vertical slope (Fig. 1) continuously growing from 0 to 1.

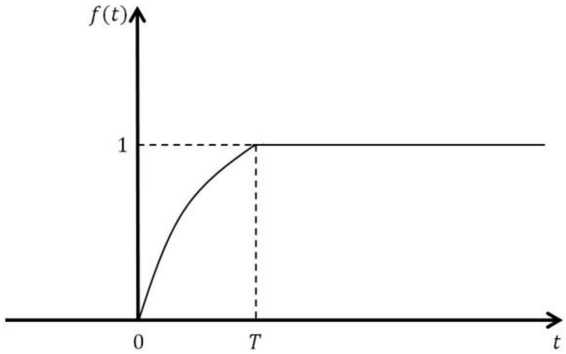

Fig. 1. Possible form of function $f(t)$.

Function $f(t)$ may be treated as a "damage accumulation" function with $f(0) = 0$ corresponding to undamaged material and $f(T) = 1$ associated with observed macroscopic fracture. The use of such function makes it possible to take into account relaxation processes at micro scale level preceding macroscopic fracture, e.g. appearance, development and coalescence of micro cracks.

Turning back to classic approach implying that stresses are relaxed instantly and follow the law $\sigma(t) = P - PH(t)$, with $P = \sigma_c$ being the ultimate stress, time to fracture t^* can be easily calculated substituting stress time dependency into fracture criterion (2) and is found to be equal to the incubation time τ. Having in mind the damage accumulation concept one can conclude that the time of stress drop T from the function $f(t)$ should be regarded as time to fracture t^* and, hence $T = t^* = \tau$. This fact gives a theoretical background for an experimental technique that can be used in order to measure the incubation time. In these experiments time history of the stress drop in the relaxation wave traveling through the sample being initiated by fracturecaused by quasistatic tensile loading should be measured. Measured time of stress relaxation should give the brittle fracture incubation time for the tested material.

The proposed experimental method involves photoelasticity methods to study the stress state of the sample. The tested PMMA is photoelastic material possessing marked birefringence properties. This means that a ray of light passing through PMMA receives two refractive indices along two principal stress directions in the stressed sample. Due to the difference between the refractive indices relative phase retardation appears between two components and hence two electromagnetic waves are produced. Optical interference of the two waves generates the fringe pattern that can be easily registered. One may establish direct relation between the fringe pattern and stress state in every point of the sample. The fringes may have different shape, thickness and color distribution. In dynamic experiments stress state is primarly given by travelling stress waves. These waves cause movement of the fringes and change of their shape.

The experimental setup consists of a standard tensile testing machine with optical measurement system mounted on it. The thin PMMA specimens had a dog-bone shape with the following dimensions:loaded zone of the specimens is 50 mm long with width and thickness equal to 10 and 5 mm correspondingly. The optical system includes helium-neon laser generating ray passing through the sample, analyzer and polarizer that is finally registered by a high quality photodiode. The photodiode generates electric signal corresponding to registered light intensity that is processed by a digital oscilloscope.

The fringe passing the measurement point (the point where the laser ray is sent) gives the change in the light intensity registered atthe photodiode. In general case the following formula can be written for the light intensity (t) [18]:

$$I(t) = kE_0^2 sin^2[2\alpha(t)] sin^2\left[\frac{\varphi(t)}{2}\right]. \quad (3)$$

Here k is a factor of proportionality, E_0 denotes the light wave amplitude, $\alpha(t)$ is the angle between the polarizer axe and the axe of one of the principal stresses. $\varphi(t)$ stands for phase retardation of the light ray transmitted through the sample. The magnitude of the relative phase retardation is given by stress-optic law [18]:

$$\varphi(t) = \frac{2\pi}{\lambda} C_\sigma[\sigma_1(t) - \sigma_2(t)]h, \quad (4)$$

where λ is the optical wave length, C_σ – the coefficient of optical sensitivity, h - the sample thickness and $\sigma_1(t)$ and $\sigma_2(t)$ are the principal stresses. Considering loading conditions realized in tensile test the stress-optic law can be significantly simplified as $\sigma_1(t) = \sigma$, $\sigma_2(t) = 0$ and $\alpha(t) = const = 45^\circ$. Therefore thelight intensity $I(t)$ should perform harmonic oscillations.

It is obvious from formulas (3) and (4) that constant stress level corresponds to constant light intensity, while the stress field alternation results in oscillations of the light intensity. Each fringe passing through measurement point is registered as a local extremum of $I(t)$.

In order to measure the time of stress relaxation (or the brittle fracture incubation time) it is necessary to distinguish the time when the stress is staring to drop from ultimate stress level and the moment of time when the stress is completely relaxed. In order to do so, one can count fringes that have passed the measurement point (extremums of light intensity) while the sample is stretched quasistatically. Before the sample is stress free in the point due topassage ofthe unloading wave the same amount of fringes should pass the measurement point. To calculate time needed for the stress to drop (incubation time τ) one should measure the time between the fringes started to move fast and the moment when all the fringes have passed the point and light intensity is almost constant.

Figure 2 gives typical *light intensity – time* dependence for long (a) and short (b) time scales. Long time scale is registered by "slow" (low frequency) oscilloscope saving data for loading stage and triggering "fast" (high frequency) oscilloscope saving unloading stage data. It was found that measured time of stress relaxation is noticeably dependent on the tensile machine crosshead movement speed. Data presented in Fig. 2 was measured for the loading rate equal to ($\dot{\sigma}(t) = 6.5\ MPa/s$). From "fast" oscilloscope measurement (Fig. 2) it can be found that for light intensity $I(t)$ it takes 17 $\mu s$to stop oscillating and therefore stress drop time is found to be equal to 17 μs for the loading rate equal to 6.5 MPa/s.

Fringe number $n(t)$ versus time dependence is shown on Fig. 3. Again two graphs with "fast" and "slow" time scales are presented. From graph 3 b) it is clear that 17 μs are needed for stress to decrease to zero, while graph a) depicts quasistatic growth of tensile stress. On both of the graphs 3 fringes passed the point where the laser beam was pointed.

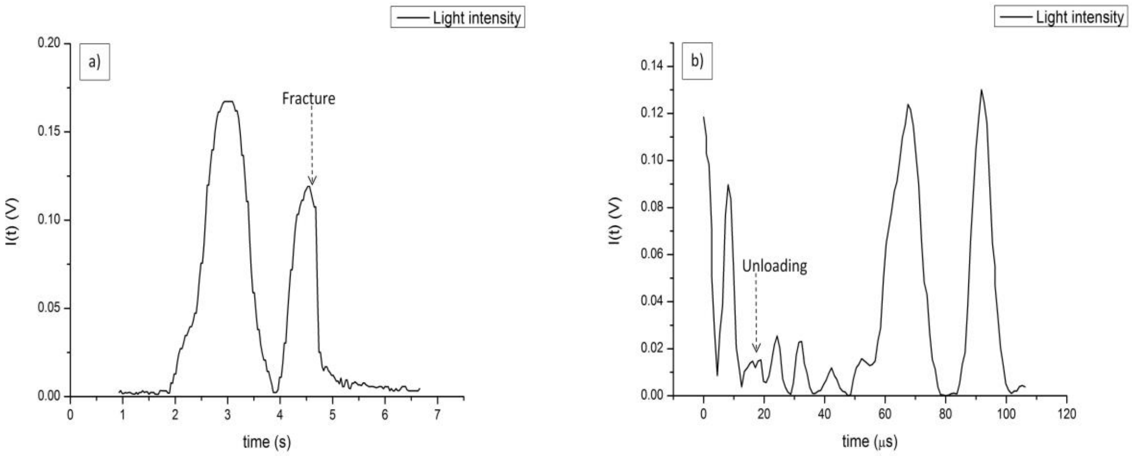

Fig. 2. "Slow" (a) and "fast" (b) time scale dependencies *Light intensity vs. time.*

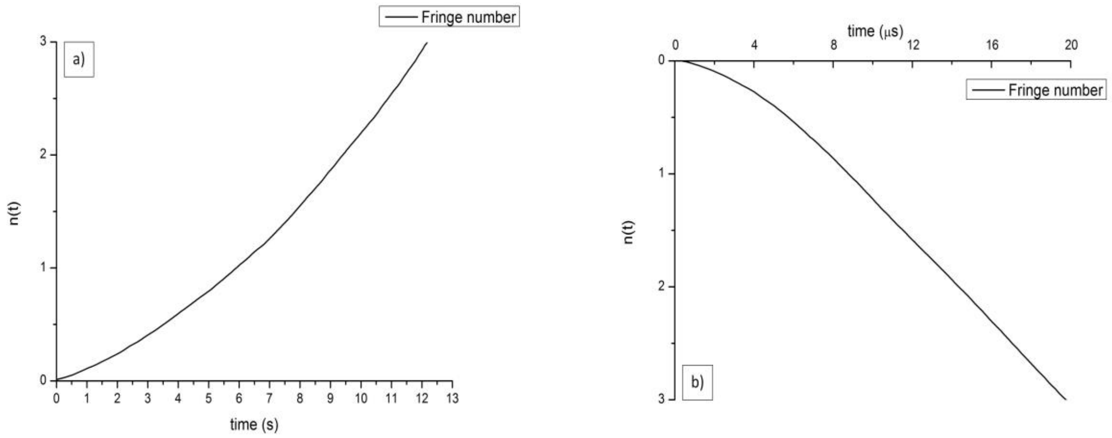

Fig. 3. "Slow" (a) and "fast" (b) time scale dependencies. *Fringe number vs. time.*

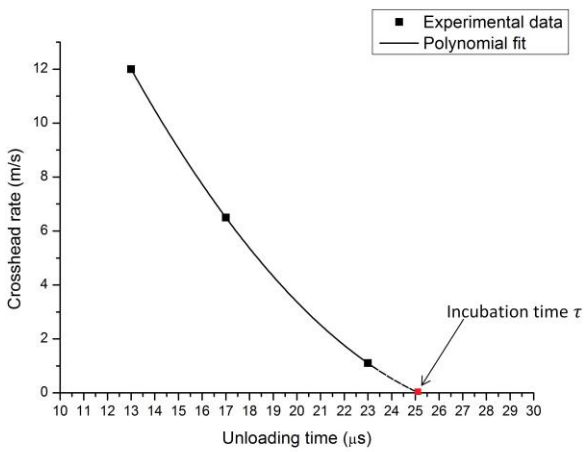

Fig. 4. *Crosshead rate vs. unloading time* dependency. Experimental points fit by 2nd order polynomial. Intersection with abscise corresponds to the incubation time.

Several tests with various crosshead movement speeds were carried out. The dependence *unloading time – crosshead rate* may be well approximated by a second order polynomial (Fig. 4). The intersection of the curve given by approximation polynomial with abscise corresponds to purely quasistatic case $\dot{\sigma}(t) = 0$ and should be treated as the measured incubation time τ for PMMA samples at a given scale level. For the performed experimental series for PMMA incubation time is found to be around 25 μs.

3. Simulation of the experiments with use of finite element method

Numerical simulationcanbe a very powerful tool in order to understand general features of dynamic processes including wave propagation pattern, fracture initialization and development points and modes [11, 12]. In many cases a computer numerical code may save a lot of time and resources as it may simulate costly experiments, help in experiment planning and partly substitute experimental work.

In this work the above-mentioned quasistatic experiments with PMMA samples were simulated and the incubation time τ was measured. The simulation was performed utilizing FEM software ANSYS in combination with an additional C++ program controlling the solution progress and fracture criterion execution.

In experiments the sample is split into two parts due to macrocrack which propagates through it. We suppose a defect presence in the sample. Simulations with a defect placed on the side of the specimen and in the specimen center were performed. The defect fails under static tensile load $\sigma < \sigma_C$. This failure generates macrocrack that propagates through the sample. While the crack is propagating the history of stresses in a point in the middle of the sample distant from the crack path (laser beam location) is traced. The simulation is over when the sample is separated into two parts.

The problem has natural symmetry and crack path coincides with line of this symmetry. This provides a possibility to consider only half of the sample constraining vertical (Fig. 5) displacements of the nodes on the crack. Releasing a particular node results in fracture propagation equal to the finite element size.

Condition leading to a particular node release is given by the incubation time fracture criterion (1). When the condition (1) is executed at some point, the corresponding node is released and hence the crack length is increased. Fracture of a single element is regarded as a micro-scale fracture and therefore in (1) the incubation time (1.5 μs) that correspondsto micro-scale levelfracture in PMMA was used for the simulation. Valuesof the incubation time for smaller scale level are determined in [19] where spall fracture was analyzed. In other words, propagation of a macrocrack is a result of numerous micro-scale fracture events. Following the incubation time approach [10] minimal crack lengthincrement is $d = 2K_{IC}^2/\pi\sigma_C^2$ and therefore element size is chosen to be equal to d.

We suppose that dynamic behavior is following linear elastic. Initial, boundary and symmetry conditions are corresponding to those realized in real experiment. The following initial boundary value problem is solved:

$$\rho \frac{\partial^2 U_i}{\partial t^2} = (\lambda + \mu)\nabla_i(\nabla \cdot \overline{U}) + \mu \Delta U_i,$$

$$\sigma_{i,j} = \delta_{i,j}\lambda \nabla \cdot \overline{U} + \mu\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right),$$

$$\overline{U}(X, t = 0) = \frac{\partial \overline{U}}{\partial t}(X, t = 0) = 0,$$

$$\sigma_{i,j}(X, t = 0) = \frac{\partial \sigma_{i,j}}{\partial t}(X, t = 0) = 0, \quad (5)$$

$U_y(X \in \Gamma_1, t) = vt,$

$U_y(X \in \Gamma_2, t) = 0$ – symmetry condition

$\sigma_y(X \in \Gamma_3, t) = \sigma_{xy}(X \in \Gamma_2 \cup \Gamma_3, t) = 0.$

Here $X = (x_1, x_2) = (x, y)$ is the coordinate, $\overline{U} = (U_1, U_2) = (U_x, U_y)$ gives the displacement vector and v is the testing machine crosshead rate. See Fig. 5 for details.

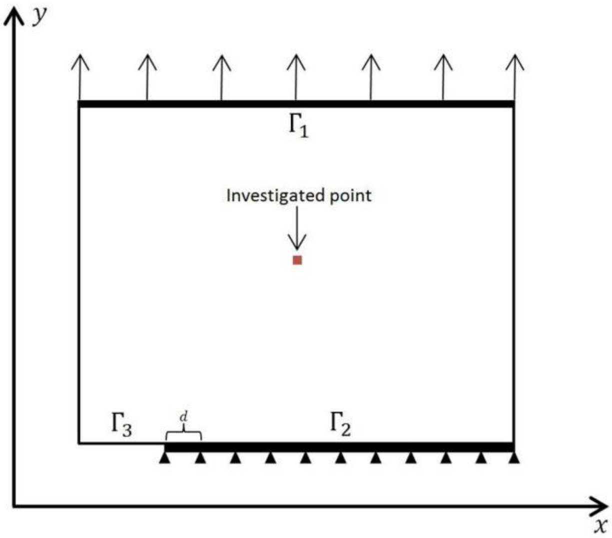

Fig. 5. Simulation scheme.

Figure 6 shows stress history in a point in the middle of the sample corresponding to the point where the stress was measured by photoelastic method. Time between the moments when the unloading wave arrives to the measurement point and when the stress is completely relaxed can be estimated from Fig. 7. It was found to be equal to 25 μs for very small crosshead movement speeds.

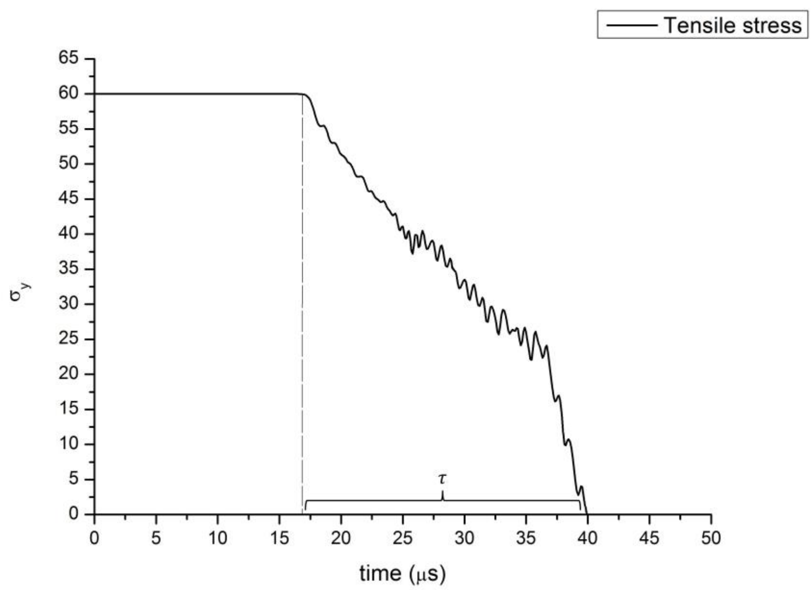

Fig. 6. Tensile stress history in the investigated point.

4. Conclusions

Incubation time is a key parameter for prediction of transient processes [13] (for example brittle fracture). In this paper a relatively simple method for incubation time of brittle fracture measurement in transparent materials with birefringence properties is proposed. The method is based on quasistatic tensile loading of a sample followed by brittle fracture. The incubation time is measured as time needed for relaxation of tensile stress at a point distant from the fracture interface. This experimental approach gives value of brittle fracture incubation time around 25 μs for thick PMMA specimens. The same result may be obtained using finite element method simulation. To simulate dynamic processes preceding macro scale fracture (in our case crack propagation) smaller scale incubation time evaluated in the case of spall fracture problem (i.e. [19]) can be utilized. This result once again [20] testifies a possibility to establish interconnection between fracture parameters on different scale levels The incubation time measured with the proposed method is very close to with the value obtained in complex and expensive purely dynamic tests (30 μs). A small discrepancy in the results of the two experimental approaches should be the topic for future investigation. One of the possible explanations can consist in considerable variation of PMMA material properties. The proposed rather simple and cheap technique can be used in order to measure incubation time of fracture in brittle transparent materials with birefringence properties. The technique can be extended to measure incubation time for brittle reflective materials or arbitrary brittle materials with thin reflective layer attached to the surface.

Acknowledgments

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