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2015 J. Phys.: Conf. Ser. 653 012049

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Numerical implementation of the incubation time fracture criterion

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Abstract. The paper is discussing problems connected with embedment of the incubation time criterion for brittle fracture into finite element computational schemes. Incubation time fracture criterion is reviewed; practical questions of its numerical implementation are extensively discussed. Several examples of how the incubation time fracture criterion can be used as fracture condition in finite element computations are given. The examples include simulations of dynamic crack propagation and arrest, impact crater formation (i.e. fracture in initially intact media), spall fracture in plates. Applicability of the approach to model initiation, development and arrest of dynamic fracture is claimed.

1. Introduction

Numerical methods are of a vital importance while solving problems of dynamic fracture mechanics. First of all this is connected to the fact that an overwhelming majority of problems of dynamic fracture are impossible to solve analytically. Framework of dynamic problems allowing analytical solution is limited to few classical solutions (e.g. see the book of Freund [1] for exhaustive collection of these solutions). Turning to problems of dynamic fracture evolution (fracture development and arrest) a possibility to construct analytical solution is completely vanishing (not accounting for couple of solutions for steady-state dynamic crack propagation).

Central issue while solving problems of dynamic fracture (no matter, numerically or analytically) is rupture criterion to be used in order to assess if fracture should happen at a given state of a system. For several decades it is known that classical fracture criteria (criteria based on the idea of the ultimate stress for intact media and on the idea of the critical stress intensity factor for cracked bodies) are not able to provide satisfactory coincidence with known experiments (see, e.g. [2]). Moreover, it is easy to show that these criteria contradict the common sense being applied to transient problems (as discussed in [3]).

In [2–4] a new criterion based on the introduced concept of the incubation time of a fracture process was proposed in order to predict conditions of initiation of brittle fracture in solids undergoing dynamic impact loading. Later in this paper the incubation time fracture criterion (ITFC) will be discussed in detail. Here some distinguishing properties of the ITFC that make it especially attractive to be embedded into numerical computational schemes are outlined.

In this connection the important feature of the incubation time fracture criterion is that it is able to predict fracture initiation conditions with reliability and correctness in “static” case of “slow” changing loads and “slow” changing geometry as well as in “dynamic” case of high-



rate loads and “fast” changing geometry (see, e.g., [2, 3]). Moreover, the criterion is supplying a smooth transition between these two cases [5]. The result is that using the approach one does not need to care about time scale of the problem—the criterion is giving correct predictions in a wide range of loading rates from static problems to the extreme dynamic ones. Even distinguishing between “static” and “dynamic” situation is not obligatory needed anymore, though the ITFC itself is providing a perfect possibility to do this.

It is easy to show (see e.g. [6]) that for “static” problems with “slow” changing loads and “slow” changing geometry the ITFC is coinciding with well-known Neuber–Novozhilov fracture criterion [7, 8]. It can be proven [3, 6, 9] that with the right choice of spatial parameter d , used in criterion formulation, Neuber–Novozhilov criterion is giving predictions coinciding with critical tensile stress (ultimate stress) criterion in the case of rupture of initially intact media and the critical stress intensity factor (Griffith–Irwin, K_{IC}) criterion in the case of rupture in a tip of a macroscopic crack. The important outcome is that the criterion is governing two cases that are normally treated separately in a single (and rather simple) rupture condition it can be applied to predict brittle fracture of materials with arbitrary size of defect, from intact undamaged media to media with macroscopic cracks. The Neuber–Novozhilov criterion is also providing smooth transition between these two cases. As a result the criterion is perfectly applicable to fracture problems with fracture surface geometry that is not known *a priori*. In such problems fracture in initially intact material can be initiated somewhere in a body and, as it evolves, transform into a macroscopic crack. The whole fracture evolution can be predicted with a single fracture criterion.

In a big number of works (see, e.g., [3, 5, 10]) authors, applying the introduced ITFC to predict critical fracture conditions in different dynamic fracture experiments (e.g., [11]) proved that the ITFC can be successfully used to predict initiation of brittle fracture appearing as a result of high-rate deformation applied somewhere in a body. In the same works a material parameter τ —the incubation time of brittle fracture, constituting the essence of the ITFC and characterizing the temporal dependence of media strength was computed for many of widely used materials.

Lately an approach making it possible to embed the ITFC into numerical computational schemes based on finite element method (FEM) was developed [16, 17]. Utilizing this approach simulation of several different experiments on dynamic impact fracture caused by high-rate loads was performed [16, 18]. These works testify that the ITFC used as a rupture criterion in FEM numerical simulations is able to predict correctly and precisely experimentally observed phenomena of dynamic fracture initiation, evolution and arrest.

As a matter of fact, not including the ITFC and approaches based on classical fracture criteria that are obviously inapplicable to predict high-rate fracture, nowadays only one approach exists that is pretending to correct prediction of dynamic fracture. This approach is originating from the works of Freund [19–21] and was later developed by Rosakes. It is based on an assumption that fracture criterion in a tip of a crack can be received as a function of stress intensity factor rate: $K^d(t) \leq K_C^d(\dot{K}(t))$, with K^d being the dynamic stress intensity factor, changing in time, K_C^d being its critical value and dot denoting time derivative. As discussed by Bratov and Petrov [16], this approach in many cases is contradicting the common sense and is applicable to predict dynamic fracture initiation (not even mentioning high-rate fracture evolution) in a very limited set of problems with strict requirements on material, loading history, fractured sample geometry etc.

The main idea of this paper is to convince the reader that the ITFC is the most promising, precise and convenient among available criteria suitable for embedding into numerical codes in order to predict dynamic fracture. The paper is also giving exhaustive information about the ITFC in connection with possibilities of its numerical implementation into FEM. The main problems of this implementation are discussed and the algorithm in order to embed the ITFC

into FEM is explicitly given.

2. Incubation time fracture criterion

Incubation time criterion for brittle fracture at a point x at time t reads as [2–4]:

$$\frac{1}{\tau} \int_{t-\tau}^t \frac{1}{d} \int_{x-d}^x \sigma(x', t') dx' dt' \geq \sigma_c, \quad (1)$$

where τ is the microstructural time of a fracture process (or fracture incubation time) a parameter characterizing the response of the studied material to applied dynamic loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse and its amplitude). d is the characteristic size of a fracture process zone and is constant for the given material and the chosen scale level. σ is normal stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress found in quasistatic conditions). x' and t' are the local coordinate and time.

Assuming

$$d = \frac{2}{\pi} \frac{K_{IC}^2}{\sigma_c^2}, \quad (2)$$

where K_{IC} is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasistatic experimental conditions. It can be shown that within the framework of linear elastic fracture mechanics for the case of fracture initiation in the tip of an existing crack, (1) is equivalent to

$$\frac{1}{\tau} \int_{t-\tau}^t K_I(t') dt' \geq K_{IC}. \quad (3)$$

Condition (2) arises from the requirement that (1) is equivalent to Irwin's criterion ($K_I \geq K_{IC}$), in the case of $t \rightarrow \infty$.

Once again it should be noticed that for slow loading rates and, hence, times to fracture that are much bigger than τ , condition (3) for crack initiation gives the same predictions as Irwin's criterion of a critical stress intensity factor. In the case when the stress field is not singular in the vicinity of point x (locally intact material) and under condition of quasistatic load applied to the media, condition (1) is reduced to critical tensile stress fracture criterion. It should be outlined that (1) in the quasistatic case is equivalent to critical stress intensity factor criterion under assumption that square root asymptotic solution is valid in the vicinity of a singular point x . In the case of a singular field that is not controlled by a square root singularity (for example asymptotic field appearing in the tip of an angular notch), when Griffith-Irwin critical stress intensity factor criterion is not applicable, condition (1) can be successfully used to predict fracture in such a singular point [22].

Thereby, (1) automatically ensures correct fracture prediction in a very wide range of quasistatic problems with materials fracturing following brittle scenario. It has been proven in multiple works (see, e.g., [2–5, 10, 23]) that for dynamic problems (1) (under condition that incubation time τ is correctly identified for the studied material) is correctly predicting stressed state at the moment of initiation of brittle rupture (in the case of fracture of initially intact media, as well as in the case of initiation of macroscopic crack). First of all this concerns problems with loads applied at high and ultra-high rates.

In this work it will be demonstrated that condition (1) can be also used to predict evolution of quasi-brittle fracture (fracture and fragmentation of initially intact media, growth and arrest of macroscopic cracks etc.).

3. Numerical implementation

Several questions are to be discussed in connection to FEM implementation of the ITFC:

- *FE mesh.* Additional requirement to FE mesh to be used in simulation with the ITFC utilized as fracture criterion consists in limitation on the size of finite elements in a vicinity of points where rupture is possible. Obviously, size of an element in this region should not exceed d (see formula (2)). Otherwise it will not be possible to perform sufficiently precise spatial integration in fracture condition (1). Also, meshing the sample, one should keep in mind that material should be separated once fracture criterion is executed somewhere in the sample. This applies both to the choice of mesh in problems without adaptive meshing (mesh is not changing throughout the simulation) and the choice of adaptive mesh that can depend on current geometry of fracture zone and other factors.
- *Time step.* In order to have a possibility to perform sufficiently precise time integration in (1) one should require that the time integration step is small as comparing to incubation time τ of the material modeled.
- *Control of fracture criterion (1) execution.* Implementation of control of fracture condition execution does strongly depend on a problem to be modeled. In some problems (for example, in the majority of problems on propagation of a macroscopic crack in unbounded media) fracture is possible only in a tip of an existing crack. In this case it is sufficient to keep track of execution condition (1) only in a single point (the tip of the crack). In other problems (for example, in the majority of problems on fracture of initially intact media) it is necessary to trace execution of (1) in rather extent zone or even in the whole modeled body. Under condition that the zone where implementation of (1) should be traced is defined and also that time step and mesh are correctly chosen, calculation of the left side in (1) does not make a big difficulty. In the examples presented later in this paper execution of (1) is controlled by an external program after every computational time step. However, with the lapse of time it is planned to create special elements for commercial FEM packages ([24,25], etc.) with criterion (1) being explicitly embedded into their formulation. Creation of such elements will considerably simplify the problem and will give a possibility to completely automate numerical simulations of brittle fracture.
- *Spatial size of defect increment (2d problems).* Incubation time theory for brittle fracture [4,23] is introducing linear size corresponding to an elementary cell of fracture on a chosen scale level. This size can also be interpreted as a typical size for the defect that one can call fracture on the chosen scale level. This size d depends on modeled material and the scale level and can be computed using (2). It makes sense to consider that once (1) is implemented in some point of the modeled body, fracture surface should be increased by the size of the elementary fracture cell d . In this connection having zones where fracture is possible meshed by elements sized d seems to be a reasonable choice.
- *Creation of a new surface.* In finite element formulation there exist several possibilities to create a new surface appearing as a result of material fracture. In the case of a crack extending along symmetry axis (in problems with symmetry) node release technique can be utilized (see, e.g., [16]). In problems without remeshing when fracture geometry is changed, node splitting technique or technique implying removal of restrictions on nodal dimensions of freedom (dof's) [18] can be used. In other situations one can use schemes assuming remeshing of the modeled body when fracture zone is changed (incremented). This approach is the most universal but at the same time the most difficult in implementation (apart from remeshing one should care about remapping of nodal values (displacements, velocities, accelerations) to new mesh). Remeshing and remapping also normally require substantial computational expense. The conclusion is that for every new problem technique to create a new surface should be specially chosen having in mind expense connected with

Table 1. Properties of Homalite-100 used in numerical simulations.

| | |
|---|------|
| Density ρ , kg/m ³ | 1230 |
| Young's modulus E , MPa | 3900 |
| Poisson's ratio ν | 0.35 |
| Critical stress intensity factor K_{IC} , MPa $\sqrt{\text{m}}$ | 0.48 |
| Ultimate tensile stress σ_c , MPa | 48 |
| Incubation time of fracture τ , μs | 9 |

model development and time needed for computations.

4. Examples

4.1. Dynamic crack propagation

To check the applicability of the criterion (1) to predict dynamic crack propagation, an attempt was made to simulate the classical fracture dynamics experiments reported by Ravi-Chandar and Knauss [11]. Detailed description of the model used in simulations and results of simulation of these experiments using FEM with the ITFC as a condition for crack extension can be found in [16]. Here some basic principles and main results are presented.

In these experiments [11] a rectangular sample with a cut simulating a crack is loaded by applying an intense load pulse to the crack faces. The sample behavior is described by equations of linear elasticity everywhere but the path of the crack, where fracture condition is given by (1). ANSYS [25] finite element package is used in order to solve linear elastic equations while implementation of (1) is controlled by an external program after each substep. The problem is symmetrical and the path of the crack is following the line of symmetry. This gives a possibility to model only half of the sample.

Nodes along the crack path are subjected to symmetrical boundary conditions up to the moment when the condition (1) is satisfied at a particular node (node movements in the vertical direction are restricted). At this moment, the restriction on movement of the particular node is removed and a new surface is created. The technique used is similar to the node release technique.

The size of elements along the crack path was taken to be exactly d (see (2)). Small elements with sizes equal to d are placed adjacent to the crack path to provide the needed accuracy of computation. Distant elements are larger in order to minimize the computational time and expense.

Material parameters typical for Homalite-100, used in the experiments of Ravi-Chandar and Knauss, were used in the calculations. These parameters are presented in table 1. The microstructural time of the fracture process, τ , for Homalite-100 was found by Petrov et al. [5] from analysis of experiments by Ravi-Chandar and Knauss [11]. The values of the critical stress intensity factor and the ultimate tensile stress gives a value for d . It appears to be 0.1 mm for Homalite-100 on a laboratory size scale.

Time profile of the load applied on the cut faces can be approximated by two consequently following trapezoids. Unfortunately, in the paper by Ravi-Chandar and Knauss there was no information about amplitude of this load. Performing multiple ANSYS computations for different amplitudes it was found that amplitudes around 5 MPa result in crack extension histories very close to those observed by Ravi-Chandar and Knauss [11]. In figure 1 the computational result for $A = 5.1$ MPa is compared to the experiments reported by Ravi-Chandar and Knauss [11].

Stress intensity factor (SIF) K_I is a key parameter, which determines stress fields around crack tip within the framework of classic linear fracture mechanics. A corresponding classic static fracture criterion is naturally extended to the case of dynamic crack propagation [12]:

$$K_I(t, P(t), \Omega(t), \dot{L}) \leq K_{Id}(K_I(t), T, \dots), \quad (4)$$

In (4) formula $P(t)$ is time-dependent loading, $\Omega(t)$ —current geometry of the specimen, $L(t)$ —crack length which changes with time, $\dot{L}(t) = dL/dt$ is current crack velocity. The right part of the expression (1) is the function called dynamic fracture toughness which is usually regarded as a material function of loading rate $\dot{K}_I(t) = \frac{dK_I}{dt}$, temperature T and other material properties. The right part of the expression (4) is supposed to be defined from experiments a priori. Such approach is widely spread in the field of dynamic fracture research. However multiple experimental results (e.g. obtained in works [11]) impugns analyses based on criterion (1) and existence of crack velocity—stress intensity factor dependence in particular. In [11] Ravi-Chandar and Kausch have shown that almost constant values of crack speed may correspond to significant change of SIF in case of explicitly dynamic sample loading. The authors of these papers supposed energy flux to the crack tip to be unrelated to crack speed, but to influence fracture surface pattern. Thus conclusions made in [11] contradict commonly applied approach based on linear fracture mechanics postulates and condition (1).

On the other hand many experimental data confirm existence of stable dependence of crack velocity $\dot{L}(t)$ on crack length $L(t)$ (which can be regarded as dependence on SIF $K_I \sim \sqrt{L}$). This effect was observed in papers [13] and [14] where experiments on thin PMMA plates are described. The experimental scheme involved quasistatic stretching of samples with an initial crack which resulted in crack acceleration followed by dynamic propagation of the crack through whole sample. Generally speaking the crack behavior observed in [13] and [14] does not contradict principles laying beneath condition (4) however one will encounter problem of determination of a functional from right part of (4) this procedure might be very expensive and complicated. Besides this classic fracture criteria similar to (4) do not consider instabilities in dependencies of fracture toughness K_{Id} on $\dot{K}(t)$.

Comparing of experiments carried out in different conditions but on the same material lets us conclude that critical stress intensity factor cannot be treated as an invariant with respect to history and conditions of loading material property which completely defines dynamic behaviour of the crack.

Classic experiments by Finberg [13] were simulated using incubation time approach. The investigators managed to monitor position of crack tip at each moment of time and therefore had an opportunity to calculate crack velocity. Figure 2 depicts dependence of crack velocity on length of the crack. Numerical results fit experimental data well [15] and therefore incubation time fracture criterion is proven to be an appropriate tool for fracture simulation in a wide range of loading conditions—from quasistatic to explosion-like loading conditions.

4.2. Impact crater formation (fracture of initially intact media)

In this section an attempt to incorporate incubation time approach into finite element (FE) code and to simulate conditions of satellite SMART1 lunar impact conducted by ESA year 2006 [26, 27] is presented. Aim of the simulation is to compare dimensions of crater created due to SMART1 contact to the moon surface to results received using FE method utilizing the ITFC as the critical rupture condition.

The traditional way to create new surface in FE formulation is associated with splitting of existing nodes. Using this approach is reasonable in most cases, though this normally requires remeshing and remapping, that are rather time consuming procedures. For the studied problem the situation is different. To guarantee correct integration in (1) one should use small (as

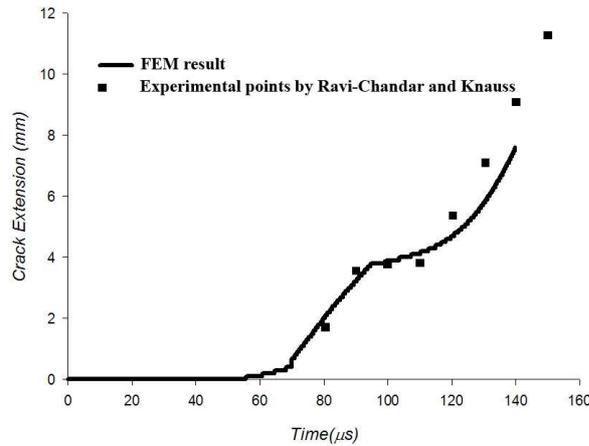


Figure 1. Crack extension history. Comparison of simulation results to experimental points received in [11].

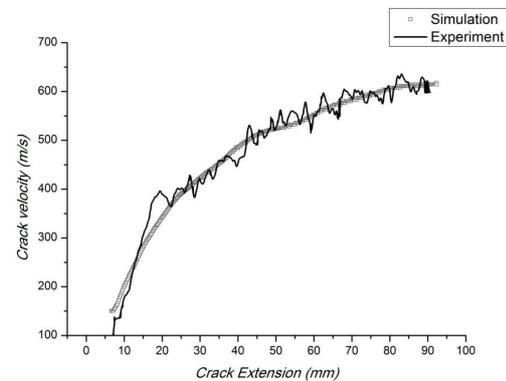


Figure 2. Crack velocity versus crack length. Comparison of simulation results to experimental data received in [13].

comparing to τ) time steps. Thus the solution is resulting in long series of tiny substeps. Solution (convergence) on every substep is achieved comparably fast FE solver is almost not iterating. It was found, that in this case it is more effective to use multiple nodes in the same location from the beginning, rather than split the node in question. Each element the full model is constructed of, is not sharing nodes with other elements.

2D problem with rotational symmetry is solved. Quadratic 4-node elements are used. Dimensions of every element is exactly d times d (where d is given by (2)). Obviously, 4 nodes have the same location for inner points of a body and 2 nodes have the same location for the points belonging to the boundary. These nodes originally have their DOF's coupled. This results in exactly the same FE solution before the fracture condition is implemented in a respective point as if elements had shared nodes. When the fracture condition is fulfilled, restriction on nodes DOF's is removed a new surface is created. This is done automatically by FE code after every substep.

Figure 3 gives a schematic representation of internal points of a body. Originally all 4 nodes sharing the same location have all of their DOF's coupled. Condition (1) for this point can be written:

$$\frac{1}{\tau} \int_{t-\tau}^t \sigma_{ii}(t') dt' \geq \sigma_c, \quad (5)$$

where i assumes values 1 and 2. Repeating indices do not dictate summation in this case. Spatial integration is removed, because the stress in the respective direction calculated by FE program is already a mean value over size d (since d is the element size being used). If (4) is fulfilled for σ_{11} and σ_{22} then displacements of nodes 1,2,3 and 4 on figure 3 get uncoupled. If (4) is fulfilled for σ_{11} , two new couple sets consisting of nodes 1, 2 and 3,4 are created. If (4) is fulfilled for σ_{22} , new couple sets are created for nodes 1, 3 and 2,4. For later times condition (4) in applicable direction is traced for newly created couple sets separately. Contact between separated fragments is not modeled which is a simplification of the simulation.

The problem is solved for half-space. Half-space representing the moon had following material properties: $\sigma_c = 10.5$ MPa, $K_{IC} = 2.94$ MPa \sqrt{m} , $\tau = 80$ μ s, $E = 60$ GPa, $\rho = 2850$ kg/m³, $\nu = 0.25$ typical for earth basalt. This results in $d = 5$ cm. Half-space is impacted by a cylinder with diameter of 1 meter and height of 1 meter. Density for the cylinder is chosen so that its mass is the same as the one of SMART1 satellite. We suppose material of cylinder is linear

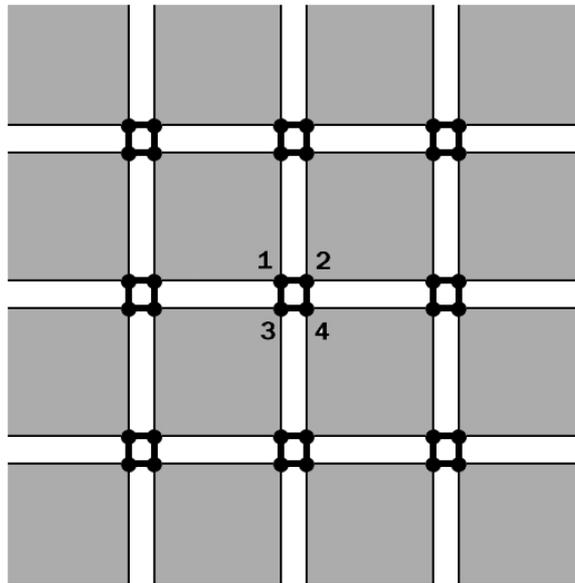


Figure 3. Model consisting of elements without shared nodes.

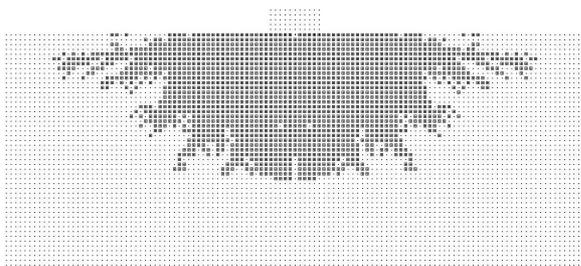


Figure 4. Locations of ruptured nodes.

elastic and has no possibility to fracture. Elastic properties are: $E = 200$ GPa, $\nu = 0.32$, typical for steel. SMART1 satellite had a form close to cubic with side of 1 meter and had a mass of 366 kg. SMART1 impacted the moon surface at a speed of approximately 2000 m/s. In FE formulation the cylinder was given an initial speed of 2000 m/s prior its contact to the half-space boundary. Size of the sample, representing the half-space is chosen from the condition that the waves reflected from the sample boundaries are not returning to the region where the crater is formed in the process of the simulation. The total of 17328 nodes and 17252 elements were used in FE model. Time step was chosen to be equal to time needed for the fastest wave to pass the distance equal to d .

ANSYS finite element package (ANSYS User's Guide [25]) was used to solve the stated problem. Control of the fracture condition (4) fulfillment in all of the sample points and new surface creation when rupture criterion is implemented was carried out by a separate ANSYS ADPL subroutine.

In figure 4, locations of nodes where the fracture occurred are marked. This gives a possibility to assess dimensions of crater that is formed after the SMART1 impact. Damaged zone is found to be about 10 meters in diameter and about 3 meters deep. Zone where the material is fully fragmented (crater formed) can be assessed having 7–10 meters in diameter and 3 meters deep. This result is coinciding with ESA estimations of dimensions of crater formed due to SMART1 impact [26, 27].

Same approach and techniques are successfully used in order to simulate spall in brittle media and impact of ceramic plates by steel plunger.

5. Conclusions

It was demonstrated that the area where the incubation time criterion for brittle fracture can be successfully used in order to simulate fracture is rather extent. An overwhelming majority of practical problems in dynamic fracture cannot be solved analytically and require numerical methods to be used in order to receive the solution. In this connection, the incubation time approach had significant advantages—it is applicable to predict both in static and dynamic fracture. Thus, there is no necessity in having different fracture criteria for different load rates. It was demonstrated that the ITFC embedded into finite element code is giving a possibility to predict initiation, development and arrest of dynamic fracture.

All this gives a reason to recommend the ITFC to be included into commercial and research FE codes as standard fracture criterion to be utilized while modeling structures that can undergo loads of the dynamic range.

Acknowledgments

The authors acknowledge Saint Petersburg State University for grant No.6.39.319.2014. The research and publication preparation was also supported by RFBR research grants No.14-01-00814 and 13-01-00598, the President Grant for Government Support of Young Russian Scientists No. MK-7596.2015.1 and the academic programs of the Russian Academy of Sciences.

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