SCIENCE CHINA
Physics, Mechanics & Astronomy

• Article •

January 2012 Vol.55 No.1: 78–85
doi: 10.1007/s11433-011-4579-3

Incubation time approach in rock fracture dynamics†

SMIRNOV V.[1], PETROV Yuri V.[2,3*] & BRATOV V.[3]

[1] St.-Petersburg State Transport University, 198103, St.-Petersburg, Russia;
[2] St.-Petersburg State University, 198504, St.-Petersburg, Russia;
[3] Institute of Problems of Mechanical Engineering, Russian Academy of Sciences, 199178, St.-Petersburg, Russia

Received October 31, 2011; accepted November 24, 2011; published online December 15, 2011

The paper is summarizing latest results connected with application of the incubation time approach to problems of dynamic fracture of rock materials. Incubation time based fracture criteria for intact media and media with cracks are discussed. Available experimental data on high rate fracture of different rock materials and incubation time based fracture criteria are used in order to evaluate critical parameters of causing fracture in these materials. Previously discovered possibility to optimize (minimize) energy input for fracture is discussed in connection to industrial rock fracture processes. It is shown that optimal value of momentum associated with critical load in order to initialize fracture in rock media does strongly depend on the incubation time and the impact duration. Existence of optimal load shapes minimizing momentum for a single fracturing impact or a sequence of periodic fracturing impacts is demonstrated.

incubation time, dynamic fracture, rocks, dynamic strength, fracture toughness, crack resistance, energy and momentum input, fracture process optimization

PACS number(s): 46.50.+a, 62.20.mm, 91.60.Ba

Citation: Smirnov V, Petrov Y V, Bratov V. Incubation time approach in rock fracture dynamics. Sci China-Phys Mech Astron, 2012, 55: 78–85, doi: 10.1007/s11433-011-4579-3

List of main symbols

$x(\{x_1,x_2\})$	coordinate
t	time
x'	local coordinate
t'	local time
τ	incubation time of a fracture
d	characteristic size of a fracture process zone
σ	normal stress
σ_c	ultimate stress
K_I	stress intensity factor for mode I loading
K_{Ic}	quasistatic limit for stress intensity factor for mode I loading (critical stress intensity factor)
ρ	mass density
λ,μ	Lame constants
u_i	displacement in direction x_i
σ_{ij}	stress in direction ij
δ_{ij}	Kronecker delta
p	pressure (stress)
P	load amplitude
t_0	load duration
$H(t)$	Heaviside step function
c_1	longitudinal wave speed
c_2	transversal wave speed
t_*	time to fracture

*Corresponding author (email: yp@yp1004.spb.edu)
†Recommended by ZHAO YaPu (Editorial Board Member)

Understanding mechanisms underlying dynamic fracture of rocks is one of the central challenges in modern rock mechanics. Dynamic loads working for fracture or fragmentation of rocks represent the essence of many industrial processes in mining and further handling of rock materials. Though for several decades it is known and generally recognized that static fracture criteria (critical stress criterion for fracture of intact media and Irwin's critical stress intensity factor criterion for fracture of cracked bodies) are not applicable to study fracture caused by dynamic loads, no conventional approach to the problem is formed to the moment.

In refs. [1,2] a new criterion to predict all the variety of experimentally observed effects typical of dynamic fracture was proposed. It was shown (refs. [2–4]) that staying within the framework of linear elastic fracture mechanics it is possible to predict all the features typical of fracture caused by high rate loads. And even more attractive is the fact, that the same critical fracture condition can be used for all load rates—from quasistatic situations, when incubation time criterion repeats classical fracture criteria, to extreme dynamic conditions, when incubation time criterion is in a very good qualitative and quantitative agreement with experimentally observed phenomena.

In this paper recent progress in application of the general incubation time approach (Petrov [5,6]) to problems of dynamic fracture of rock materials is presented.

1 Incubation time fracture criterion

Incubation time fracture criterion, originally proposed to predict crack initiation in dynamic conditions, was formulated in refs. [1,3]. This criterion for failure at a point *x*, at time *t*, is expressed as:

$$\frac{1}{\tau}\frac{1}{d}\int_{x-d}^{x}\int_{t-\tau}^{t}\sigma\left(x',t'\right)\mathrm{d}x'\mathrm{d}t' \leqslant \sigma_{\mathrm{c}}, \tag{1}$$

where τ is the incubation time of a fracture process (or microstructural time of fracture)—a parameter characterizing the response of the material to applied dynamic loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse or its amplitude). d is the characteristic size of a fracture process zone and is constant for the given material and chosen spatial scale. σ is normal stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress evaluated in quasistatic conditions). x' and t' are local coordinate and time.

Assuming

$$d = \frac{2K_{\mathrm{Ic}}^{2}}{\pi\sigma_{\mathrm{c}}^{2}}, \tag{2}$$

where K_{Ic} is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasistatic experimental conditions, it can be shown that within the framework of linear fracture mechanics for case of fracture initiation in the tip of an existing mode I loaded crack, eq. (1) is equivalent to:

$$\frac{1}{\tau}\int_{t-\tau}^{t}K_{\mathrm{I}}\left(t'\right)\,\mathrm{d}t' \leqslant K_{\mathrm{Ic}}. \tag{3}$$

Condition (2) arises from the requirement that eq. (1) is equivalent to Irwin's criterion $K_{\mathrm{I}} \leqslant K_{\mathrm{Ic}}$ in quasistatic conditions $(t_0 / \tau \to \infty)$. This means that a certain size typical of fractured material appears. This size should be associated with a size of a failure cell on the current spatial scale – all rupture sized essentially less than d cannot be called fracture on the current scale level. At the same time smaller scale fracture can happen, but is supposed to be not essential for observed fracture processes.

Thus, time-spatial domain is "quantized" by means of introduction of τ and d. Once the material and the scale are chosen, τ gives a time, such, that a portion of momentum (or energy) accumulated during this time can be released by rupture of the corresponding cell [5,6]. Linear size d assigns dimensions for the cell. Introduction of time and spatial discretization is a very important step. To our belief, a correct description of high loading rate effects is not possible if this time-spatial discreteness is not accounted somehow. Advantage of the incubation time approach is that one can stay within the framework of linear elasticity, utilizing all the consequent advantages. Discreetness of the problem is accounted only by the special form of the critical fracture condition.

As shown in multiple publications (refs. [4,5,7]), criterion (3) can be successfully used in order to predict fracture initiation and structural transformations in brittle solids. For slow loading rates and, hence, times to fracture that are essentially bigger than τ, condition (3) for crack initiation gives the same predictions as Irwin's criterion of the critical stress intensity factor [8]. For high loading rates and times to fracture comparable with τ all the variety of effects experimentally observed in dynamic fracture experiments (refs. [9–12]) can be received using eq. (3), both qualitatively and quantitatively [6]. Application of condition (3) in models describing real experiments or usage of eq. (3) as critical fracture condition in finite element numerical analysis gives a possibility for better understanding of fracture dynamics nature (refs. [13,14]) and even prediction of new effects typical of dynamic processes (ref. [15]).

Another known approach to dynamic fracture, originating from works of Freund [16] and later developed by Freund [17] and Rosakis (for ref. [18]) is based on the assumption that dynamic fracture toughness can be directly and unequivocally coupled with the loading rate or the stress intensity factor rate. Sometimes, for specific experimental conditions when the stress intensity factor (or stress) is monotonously growing with time, such a dependency can be observed in reality. But generally speaking the majority of known experimental results for high rate fracture stand for inapplicability of this approach. Both strength rate dependency and fracture toughness rate dependency are characterized by extremely unstable behavior. Moreover, in numerous experiments [19–21] it is observed, that fracture can initiate at a moment when the stress intensity factor (or local stress, if concerning fracture of initially intact material, for example, in dynamic cleavage experiments) is decreasing and, hence, is having negative rate. Obviously these phenomena are impossible to predict presuming unequivocal dependency of fracture toughness (or critical stress) on stress intensity factor rate (or stress rate).

It was shown that using the incubation time criterion incorporated into finite element code a correct prediction of dynamic fracture initiation [2,5], dynamic crack propagation [22] and fracture of initially fractured media [23] is possible.

All this is giving reasons to conclude that the incubation time based approach in dynamic fracture has the most potential among currently known approaches.

In order to utilize the incubation time approach for analysis of rock materials one needs to determine incubation process characteristics for particular rocks. Experiments on dynamic fracture of rock specimens were carried out at Research Center "Dynamics" of the St. Petersburg State University. Dynamic loading was created by magnetic field using experimental equipment developed by Krivosheev and Petrov [24]. An approach based on the incubation time concept was used to evaluate dynamic fracture toughness of the material.

Data presented in Table 1 was experimentally evaluated by Petrov et al. [25], incubation time τ was found by analysis of threshold amplitudes of high-rate loads [24], parameter d is calculated utilizing eq. (2).

Threshold (minimum fracturing) amplitudes for microsecond-range loads applied to faces of preexisting crack in plates made of different rocks were determined. Specimen sizes were "200 mm×200 mm×12 mm" for gabbro-diabase, "100 mm×100 mm×25 mm" for limestone, "300 mm×300 mm×10 mm" for granite, "120 mm×120 mm×30 mm" for clay, "163 mm×163 mm×20 mm" for sandstone and "156 mm×156 mm×20 mm" for marble respectively. Static mechanical properties for these materials were evaluated from data obtained in tests using standard material testing equipment.

Further we summarize some results connected with application of the incubation time approach to problems of dynamic fracture of rock materials. Incubation time based fracture criteria for intact media and media with cracks are discussed. A possibility to control external high-rate impact in order to optimize energy input for fracture of some of the rock materials is studied. It is shown that minimum energy in order to initialize fracture in cracked rock media does strongly depend on amplitude and duration of an impact causing this rupture. Existence of optimal energy saving shapes for a single impact or a sequence of periodic impacts is demonstrated.

2 Spall strength of rocks

In case of "intact" (defect-free) media fracture criterion (1) can be rewritten:

$$\int_{t-\tau}^{t} \sigma(s)\,\mathrm{d}s \leqslant \sigma_{\mathrm{c}}\tau. \tag{4}$$

Consider compressive triangularly symmetric shaped wave traveling along semi-infinite rod:

$$\sigma_{-}(x,t) = -P\left\{\frac{ct+x}{ct_0}\left[H(ct+x) - H(ct+x-ct_0)\right]\right.$$
$$\left. + \left(2 - \frac{ct+x}{ct_0}\right)\left[H(ct+x-ct_0) - H(ct+x-2ct_0)\right]\right\},$$

where P is giving the pulse amplitude, $2t_0$ is the load duration, $H(t)$ is the Heaviside step function and c is the sound speed. The wave is reflecting from the stress-free end (x=0) of the rod and its sign is changed from compression to tension:

$$\sigma_{+}(x,t) = +P\left\{\frac{ct-x}{ct_0}\left[H(ct-x) - H(ct-x-ct_0)\right]\right.$$
$$\left. + \left(2 - \frac{ct-x}{ct_0}\right)\left[H(ct-x-ct_0) - H(ct-x-2ct_0)\right]\right\}.$$

The resulting stress in the rod is given by: $\sigma(x,t) = \sigma_{-}(x,t) + \sigma_{+}(x,t)$. Obviously, maximum tensile stress firstly appears at the point x_0=ct_0/2. Introducing dimensionless variables: $T = t/\tau$, $T_0 = t_0/\tau$, one can receive:

Table 1 Properties of some rock materials

N	Rock	σ_{c} (MPa)	K_{Ic} (MPa$\sqrt{\mathrm{m}}$)	d (mm)	τ (μs)
1	Limestone	12.40	1.31	7.11	15
2	Gabbro-diabase	44.04	2.36	1.83	40
3	Marble	6.19	1.34	30.00	44
4	Sandstone	31.18	1.19	0.93	54
5	Granite	19.50	1.08	1.95	69
6	Clay	1.63	0.12	3.45	75

$$\sigma(T)\Big|_{x=x_0} = F(T)+G(T); \tag{5}$$

$$F(T) = -P\left\{\left(\frac{1}{2}+\frac{T}{T_0}\right)\left[H\left(T+\frac{T_0}{2}\right)-H\left(T-\frac{T_0}{2}\right)\right] + \left(\frac{3}{2}-\frac{T}{T_0}\right)\left[H\left(T-\frac{T_0}{2}\right)-H\left(T-\frac{3T_0}{2}\right)\right]\right\};$$

$$G(T) = +P\left\{\left(\frac{T}{T_0}-\frac{1}{2}\right)\left[H\left(T-\frac{T_0}{2}\right)-H\left(T-\frac{3T_0}{2}\right)\right] + \left(\frac{5}{2}-\frac{T}{T_0}\right)\left[H\left(T-\frac{3T_0}{2}\right)-H\left(T-\frac{5T_0}{2}\right)\right]\right\}.$$

Threshold (minimum) amplitude P_*, leading to fracture in the rod can be found utilizing fracture criterion (4) for any given duration t_0:

$$\max_T I(T) = \sigma_c, \quad I(T) = \int_{T-1}^{T} \sigma(s)\,\mathrm{d}s. \tag{6}$$

Obviously:

$$\max_T I(T) = I\left(\frac{3T_0}{2}+\frac{2}{3}\right) \quad \text{at} \quad T_0 \geqslant \frac{2}{3};$$

$$\max_T I(T) = I\left(T_0+1\right) \quad \text{at} \quad T_0 \leqslant \frac{2}{3},$$

i.e., time to fracture T_* can be calculated as:

$$T_* = \frac{3T_0}{2}+\frac{2}{3} \quad \text{at} \quad T_0 \geqslant \frac{2}{3}; \qquad T_* = T_0+1 \quad \text{at} \quad T_0 \leqslant \frac{2}{3}, \tag{7}$$

and

$$\max_T I(T) = I\left(T_*\right) = P\left(1-\frac{1}{3T_0}\right) \quad \text{at} \quad T_0 \geqslant \frac{2}{3}; \qquad \max_T I(T) = I\left(T_*\right) = \frac{3}{4}PT_0 \quad \text{at} \quad T_0 \leqslant \frac{2}{3}. \tag{8}$$

Now, using eqs. (6) and (7) one can determine time to fracture T_* as a function of the threshold amplitude P_*

$$T_*(P_*) = \begin{cases} \dfrac{1}{2\left(1-\sigma_c / P_*\right)}+\dfrac{2}{3}, & \text{at} \quad 1 \leqslant \dfrac{P_*}{\sigma_c} \leqslant 2; \\ \dfrac{4\sigma_c}{3P_*}+1, & \text{at} \quad \dfrac{P_*}{\sigma_c} \geqslant 2. \end{cases} \tag{8}$$

In dimensional variables:

$$t_* = \begin{cases} \dfrac{3}{2}t_0+\dfrac{2}{3}\tau, & \text{at} \quad t_0 \geqslant \dfrac{2}{3}\tau; \\ t_0+\tau, & \text{at} \quad t_0 \leqslant \dfrac{2}{3}\tau. \end{cases} \tag{9}$$

It is seen from the second expression in eq. (9) that $t_* \to \tau$ as $t_0 \to 0$. Thus, the incubation time τ is the time to specimen fracture t_* while it is loaded by threshold pulse of infinitesimal duration (i.e., by pulse having Dirac delta-function form). At threshold loads (with amplitudes equal to P_*) time to fracture cannot be shorter than τ, a certain period of time (incubation time) is needed for the material “to prepare” fracture. Time to fracture can be less than incubation time only in case of over threshold loads, i.e. at overloaded impacts.

Analysis of temporal strength dependence gives a possibility to draw important conclusions about interrelation and evidence variety of quasistatic and dynamic spall fracture mechanisms. The resultant diagram of temporal strength dependence (Figure 1) is the main characteristic of spall strength. It is evident that the fracture threshold is essentially determined by both the dynamic fracture parameter τ and the static strength of material. The static branch (long loads, low threshold amplitudes) is fully controlled by the static material strength σ_c, while the dynamic branch (short loads, higher threshold amplitudes) is mainly controlled by the fracture incubation time τ. As it follows from Figure 1 even though gabbro-diabase has larger quasistatic tensile strength, in conditions of high-rate loading it appears to be easier to fracture as compared to sandstone.

One can calculate (threshold) momentum corresponding to the threshold loads leading to spall fracture. It can be calculated as $U_*(t_0) = P_* t_0$ for the studied time shape of the load (isosceles triangle). Threshold amplitude P_* can be found from eqs. (5) and (7):

$$P_*(t_0) = \begin{cases} \dfrac{\sigma_c}{1-\dfrac{\tau}{3t_0}}, & \text{at} \quad t_0 \geqslant \dfrac{2}{3}\tau; \\ \dfrac{4\sigma_c}{3t_0}\tau, & \text{at} \quad t_0 \leqslant \dfrac{2}{3}\tau. \end{cases}$$

Then the threshold momentum leading to fracture will be:

$$U_*(t_0) = \begin{cases} \dfrac{3}{3}\dfrac{\sigma_c\, t_0^2}{t_0-\tau}, & \text{at} \quad t_0 \geqslant \dfrac{2}{3}\tau; \\ \dfrac{4}{3}\sigma_c\tau, & \text{at} \quad t_0 \leqslant \dfrac{2}{3}\tau. \end{cases} \tag{10}$$

The upper equation in eq. (10) is controlling the static branch of strength while the lower one is giving the dynamic branch. Figure 2 is the graphical representation of $U_*(2t_0)$ for the above-mentioned rocks.

One more threshold strength characteristic can be received as a product of threshold amplitude P_* and time-to-fracture t_* : $G = P_* t_*$. This value will be here referred to as pulse capacity of fracture (PCF)—see sect. 4 for more details. Figures 3 and 4 present dependencies of $G(t_0)$ on the load duration.

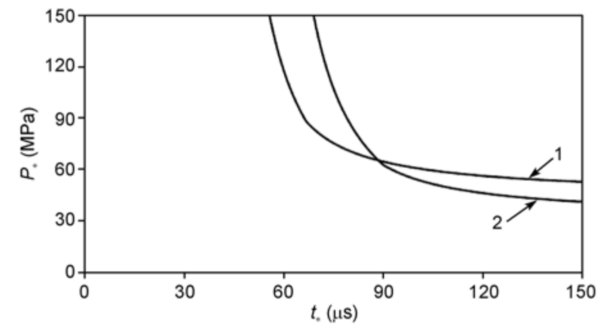

Figure 1 Temporal spall strength dependence of rocks. 1-gabbro-diabase, 2-sandstone.

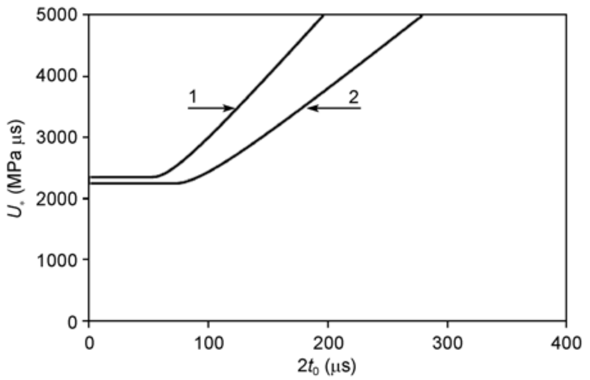

Figure 2 Threshold fracture momentum. 1-gabbro-diabase, 2-sandstone.

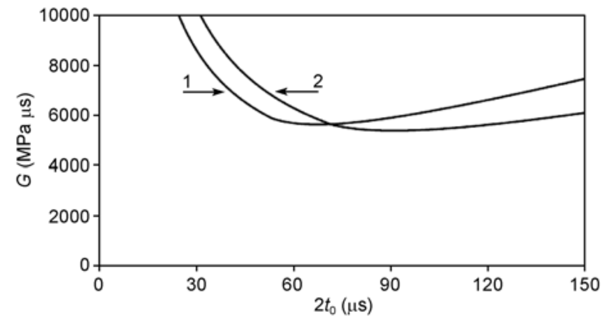

Figure 3 PCF versus load duration ($0\leqslant 2t_0\leqslant 150$ μs). 1-gabbro-diabase, 2-sandstone.

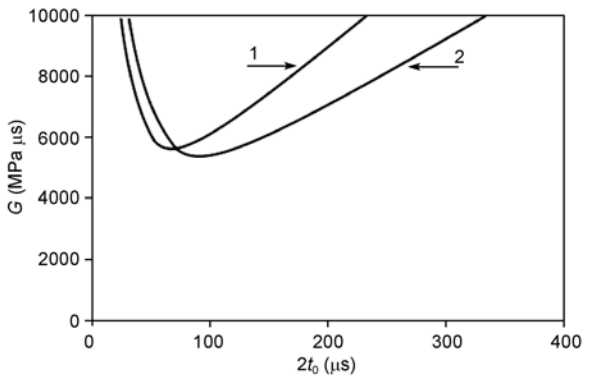

Figure 4 PCF versus load duration ($0\leqslant 2t_0\leqslant 400$ μs). 1-gabbro-diabase, 2-sandstone.

It is remarkable that the presented dependencies display a marked minimum. This is due to the fact that the time-to-fracture t_* increases as the load duration is growing (Figure 5) while the threshold load amplitude is decreasing (Figure 6). The above-mentioned fact is giving a possibility to predict optimal energy-saving parameters for processes working for fracture of rock materials. For example, for gabbro-diabase optimal load duration is equal to 66 μs and for sandstone 92 μs. Minimum PCF $G_{\min}$ for these rocks are 5627 and 5378 MPa μs correspondingly.

3 Prediction of dynamic fracture toughness for rock materials

An important conclusion from the previous section is, that in order to use incubation time fracture criterion for practical predictions of critical rupture conditions one should supplement static material specific strength parameters (ultimate stress σ_c and critical stress intensity factor K_{Ic}), that are known for majority of rock materials, with incubation time of the fracture process for material in question (τ). In this section the theoretical background for one class of experiments aimed for evaluation of τ is given and corresponding experimental results for rock materials are presented.

An infinite plane with a semi infinite crack ($\{x_1,x_2\}$:x_2=0,

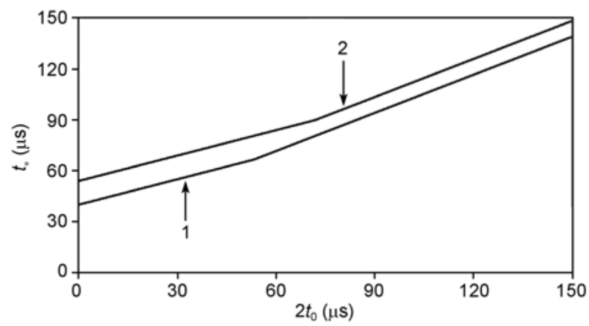

Figure 5 Time-to-fracture versus load duration. 1-gabbro-diabase, 2-sandstone.

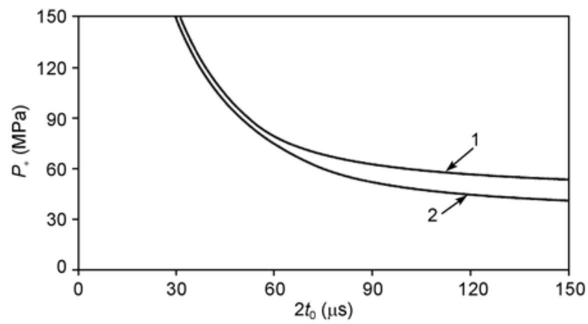

Figure 6 Threshold fracture amplitude versus load duration. 1-gabbro-diabase, 2-sandstone.

x_1<0) is considered. Plane strain conditions are supposed. The load is given as a pressure pulse applied on the crack faces. Displacements of the plane are described by

$$\rho u_{i,tt} = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj}, \tag{11}$$

where "," refers to the partial derivative with respect to time and spatial coordinates. ρ is the mass density, and the indices i and j assume the values 1 and 2. Displacements are given by u_i in the directions x_i respectively. t stands for time, λ and μ are Lame constants. Stresses and strains are coupled by the Hooke's law:

$$\sigma_{ij} = \lambda \delta_{ij} u_{k,k} + \mu (u_{i,j} + u_{j,i}), \tag{12}$$

where σ_{ij} represents stresses in direction ij, δ_{ij} is the Kronecker delta assuming value of 1 for $i=j$ and 0 otherwise. For negative times the plane is stress free and at rest:

$$\sigma_{ij}\big|_{t<0} = 0, \quad u_{,t}\big|_{t<0} = 0. \tag{13}$$

Crack faces are free from tractions:

$$\sigma_{21}\big|_{x_1<0,\, x_2=0} = 0. \tag{14}$$

Load on the crack faces is given by:

$$\sigma_{22}\big|_{x_1<0, x_2=0} = -p(t). \tag{15}$$

It is assumed that the leading term in Williams asymptotic expansion near crack tip stresses is controlling the stress field ahead of the crack:

$$\sigma_{22}\big|_{x_1>0,\, x_2=0} = \frac{K_I(t)}{\sqrt{2\pi x_1}} + O(1), \quad x_1 \to 0. \tag{16}$$

Rectangular shaped load pulse is applied on the crack faces:

$$p(t) = P\left[H(t) - H(t - t_0)\right], \tag{17}$$

where P and t_0 prescribe amplitude and duration for the load pulse and $H(t)$denotes the Heaviside step function. Solving eqs. (11)–(17) one can find stress intensity factor history:

$$K_{\mathrm{I}}(t) = P\varphi(c_1, c_2) \left[\sqrt{t}\ H(t) - \sqrt{t - t_0}\ H(t - t_0)\right], \tag{18}$$

where

$$\varphi(c_1, c_2) = \frac{4\ c_2 \sqrt{c_1^2 - c_2^2}}{c_1 \sqrt{\pi\ c_1}},$$

with c_1 and c_2 being the speeds of longitudinal and transversal wave in the studied material, respectively.

Supposing that the amplitude of the load is the threshold one (i.e., the minimal possible amplitude resulting in crack extension), time to fracture following (3) can be found from:

$$\max_t I(t) = K_{\mathrm{Ic}} \tau, \quad I(t) = \int_{t-\tau}^{t} K_{\mathrm{I}}(t')\, \mathrm{d}t'. \tag{19}$$

Substituting K_{I} from eq. (18) into eq. (19) one can get:

$$\begin{aligned} I(t) = &\frac{2}{3} P\varphi\ (c_1, c_2) \\ &\times\left[t^{3/2} H(t) - (t-\tau)^{3/2} H(t-\tau) - (t-t_0)^{3/2} H(t-t_0)\right. \\ &\left. + (t-\tau-t_0)^{3/2} H(t-\tau-t_0)\right]. \end{aligned} \tag{20}$$

Obviously $I(t)$ reaches its maximum overtime value at $t = t_*$:

$$t_* = \frac{1}{3}\left[\ \tau + t_0 + 2\ \sqrt{\tau^2 - \tau\ t_0 + t_0^2}\ \right]. \tag{21}$$

Thus, conducting series of experiments on cracked plates with sizes such, that the waves from the specimen boundaries are not reaching crack tip prior to crack initiation, tending to find the threshold load amplitude for pulses of a given duration t_0, one can obtain the incubation time τ for the tested material.

4 Optimization of energy input in industrial processes connected with fracture of rock materials

An approach described in the previous section was applied to evaluate structural parameters and critical characteristics of dynamic fracture of particular rock materials. Table 1 gives the values for critical stress intensity factor, ultimate stress and incubation time for several rock materials.

Figure 7 reflects dependencies of time-to-fracture t_* on a threshold pulse duration t_0 for such materials. Presented curves were computed using the incubation time criterion with material parameters taken from Table 1.

A possibility to reduce the amount of energy, required for fracture or fragmentation of rock materials is of a great importance for mining and rock processing technologies.

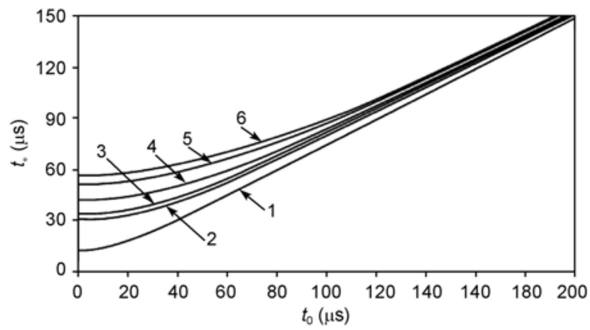

Figure 7 Time-to-fracture—load duration curves for rock materials. 1-limestone, 2-gabbro-diabase, 3-marble, 4-sandstone, 5-granite, 6-clay.

Examples applications include such areas as percussive, explosive, hydraulic, electro-impulse and other mining technologies as well as drilling, pounding, etc. In these processes energy input often accounts for a significant, if not the largest, part of the process cost (see for ref. [26]). By different estimations up to 25% of all energy produced on our planet is spent for fracture.

For practical problems it is often convenient to have a dependence of some integral (general) strength parameter on independent (controllable in experiment) variable (for example t_0, being the load duration). For some applications it can be useful to assess momentum transferred to the loaded media in the process of impact interaction. Threshold momentum, called here the pulse capacity of fracture (PCF), can be related to the product of threshold fracture amplitude value and time to fracture $G=P_* t_*$. One can calculate the dependence of G on the loading duration t_0. Once this is done, one will see that some definite load duration is giving a minimum $G_{\min}$ for $G(t_0)$. The existence of this minimum is explained by the fact that time to fracture t_* should increase and the threshold load amplitude P_* should decrease as the load duration is increased. Finding the loading pulse giving the minimum for G should also give the loading pulse minimizing momentum that should be transferred in order to produce fracture of this material.

PCF is the characteristic of material strength in dynamic conditions which possesses the following advantages:

(1) Quantitative estimation of G is normally not making any difficulty: critical load amplitude P_* can be directly measured in experiment and the time-to-fracture t_* can be easily calculated (using equation similar to eq. (21));

(2) Value for G is the threshold characteristic, i.e. it is providing a combination of minimum load parameters for creating fracture in the material. It is known that for threshold loads special structural-temporal features of dynamic fracture are becoming specially apparent;

(3) PCF is physically more substantive as compared to energy as this characteristic is accounting for the type of the load applied (tension/compression).

Figure 8 shows the variation of $G(t_0)$ for rectangular shape load pulse (17). Thus for example it was found that for limestone the optimal (minimizing momentum) fracturing load duration is equal to 20.0 μs. For sandstone it was found to be 74.3 μs (Figure 8). Minimum possible values $G_{\min}$ for these rocks are 129 and 193.2 MPa μs correspondingly. At the optimal load duration it is gabbro-diabase that has proved to be the strongest material among all six rocks while clay is one of the least in its strength.

In ref. [15] amount of energy sufficient to initiate propagation of a mode I loaded central crack was extensively explored. It was demonstrated that energy to input into media containing a crack in order to advance this crack does strongly depend on the shape of the load pulse applied. The most important conclusion is the existence of optimal energy saving parameters for fracturing machines. Controlling

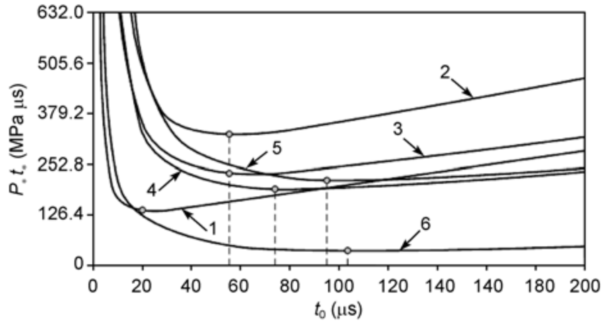

Figure 8 Dependence pulse capacity of fracture on load duration. 1-limestone, 2-gabbro-diabase, 3-marble, 4-sandstone, 5-granite, 6-clay.

amplitude and duration (frequency) of impacts created by a machine it is possible to reduce energy spent on creation of the desired rupture.

The majority of nonexplosive industrial processes connected with fracturing or fragmenting of rock materials provide a possibility to control amplitude and frequency of the created impacts. There is a possibility to optimize energy input for fracture and fragmentation in such processes by adjusting the amplitude and the frequency of a rupture machine impacts. Prediction of optimal energy saving parameters can be done on the basis of material properties including elastic parameters, strength properties, incubation time of fracture, and information about the prevalent size and distribution of defects in fractured material.

It can be demonstrated that for majority of rock materials application of fracturing impacts with parameters significantly different from the optimal ones, requires much more energy compared to application of optimal energy saving impacts resulting in similar fracture. For example for granite with initial cracks of 5 mm fractured by rectangular shaped load pulses, if the frequency of the fracturing impacts differs from the optimal one by 10%, energy cost of fracture creation is exceeding the minimal value by 12% [15]. Taking into consideration the fact that the efficiency of rupture connected processes rarely exceeds a few percent, it gets evident that even a possibility of small improvement of energy input is of a great importance for industry.

5 Conclusions

The central problem of testing the dynamic strength properties of rocks can be associated with measurements of the incubation time parameter. Studies of threshold characteristics (pulse amplitudes, time to fracture, etc.) of fracture processes provide an effective opportunity to examine the incubation stage of the fracture process and to evaluate a set of fixed material parameters for structural-time criterion. Different experiments (i.e., spall fracture, experiments on dynamic fracture toughness, etc.) can be interpreted within the framework of a single theory using the incubation time approach. Obviously any of these experimental schemes can be used for independent dynamic testing of materials.

Calculations based on the incubation time concept indicate a possibility to optimize fracture of rock materials caused by dynamic loads. This optimization can be attained by a proper choice of the load parameters. Two principal characteristics of dynamic fracture: threshold amplitude and time-to-fracture associated with particular loads can be determined on the basis of the incubation time criterion introduced into the corresponding dynamic problem solution. It is proved that the value of a product of these parameters (pulse capacity of fracture) can be effectively optimized for particular materials.

This work was supported by RFBR Research (Grant Nos. 10-01-00810-a, 11-01-00491-a and 10-01-91154-GFEN_a), Russian Federation State Contracts and Academic Programs of the Russian Academy of Sciences.

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