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Spatio-temporal Model of Multiscale Fracture in Brittle Solids

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Abstract: We analyze examples illustrating typical dynamic effects inherent in dynamic fracture process, and we propose a unified interpretation for fracture of solids utilizing structural-temporal approach based on the concept of the fracture incubation time. Corresponding generalized model accounting for fracture scale level will also be presented. The model will be used to predict fracture of quasi-brittle heterogeneous materials on different scale levels. It will be demonstrated that can give a possibility to predict fracture on a higher (real) scale level and having experimental data obtained on a lower (laboratory) scale level.

Key words: incubation time criterion; dynamic fracture; dynamic strength; fracture toughness; spatio-temporal discretization; scale levels; crack propagation; pipelines; FEM modelling.

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Experiments on the dynamic fracture of solids caused by intense dynamic loads or focused energy fluxes reveal a number of effects indicating fundamental differences between dynamic rupture (breakdown) of materials and a similar process under slow quasistatic loads. For example, one of the basic problems in testing of dynamic-strength properties of materials is associated with the dependence of the limiting characteristics on the duration, amplitude, and rate of an external load, as well as on a number of other factors. Whereas a critical value is a constant for a material in the static case, experimentally determined critical characteristics in dynamics are strongly unstable, and as a result, their behavior becomes unpredictable. The indicated (some other) features of the behavior of materials subjected to pulsed loads are common for a number of seemingly quite different physical processes, such as dynamic fracture (crack initiation and scabbing), cavitation in liquids, and electrical breakdown in solids. In this paper, we analyze examples illustrating typical dynamic effects inherent in dynamic fracture of brittle media. We propose a unified interpretation for the fracture of solids using the structural-temporal approach based on the concept of the fracture incubation time [1-4]. This paper is also presenting structural-temporal approach for analysis of multiscale nature of brittle fracture. Problem of experimental determination of a fixed scale level is discussed. Possible interconnections of this scale level with a higher and a lower scale levels are discussed.

1　Incubation-time criterion

The basic cause of difficulties in modeling the aforementioned effects of mechanical strength is the absence of an adequate limiting condition that determines the instant of rupture or breakdown. This problem can be solved using both the structural macromechanics of fracture and the concept of the fracture incubation time,

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which represents the kinetic processes of the formation of macroscopic breaks [1-3]. The above effects become essential for loads with periods comparable to the scale determined by the fracture incubation time associated with preparatory relaxation processes of microdefects development in the material structure.

The criterion for the fracture incubation time proposed in [1-4] makes it possible to calculate effects of the unstable behavior of dynamic-strength characteristics. These effects are observed in experiments on the fracture of solids. This criterion can be generalized in the form of the condition:

$$\frac{1}{\tau}\cdot\int_{t-\tau}^{t}\left(\frac{F(t')}{F_c}\right)^{\alpha}\mathrm{d}t'\leqslant 1, \tag{1}$$

where $F(t)$ is the intensity of a local force field causing the fracture of the medium, F_c is the static limit of the local force field, and τ is the incubation time associated with the dynamics of a relaxation process preparing the break. The fracture time t_* is defined as the time at which condition (1) becomes an equality. The parameter α characterizes the sensitivity of a material to the intensity of the force field causing fracture.

By example of the mechanical break of a material, we now consider one of the possible methods of interpreting and determining the parameter τ. We assume that a standard sample made of a given material under tension is broken into two parts under the stress P arising at a certain time $t=0: F(t)=PH(t)$, where $H(t)$ is the heaviside step function. In the case of quasi-brittle fracture, the material is unloaded, and the local stress at the break point decreases rapidly (but not instantaneously) from P to 0. In this case, the corresponding unloading wave is generated, propagates over the sample, and can be detected by well-known (e.g., interferometric) methods. The stress variation at the break point can be conditionally represented by the dependence $\sigma(t)=P-Pf(t)$, where the function $f(t)$ varies from 0 to 1 within a certain time interval T. The case $f(t)=H(t)$ corresponds to the classical theory of strength. In other words, according to the classical approach, break occurs instantaneously ($T=0$). In practice, the break of a material (sample) is a process proceeding in time, and the function $f(t)$ describes the micro-scale level kinetics of the passage from a conditionally defect-free state ($f(0)=0$) to the completely broken state at the given point ($f(0)=1$) that can be associated with the macro-fracture event (Fig. 1). On the other hand, applying fracture criterion (1) to macro-scale level situation $(F(t)=PH(t))$, we arrive at the relation for time to fracture $t_*=T=\tau$ for $P=F_c$.

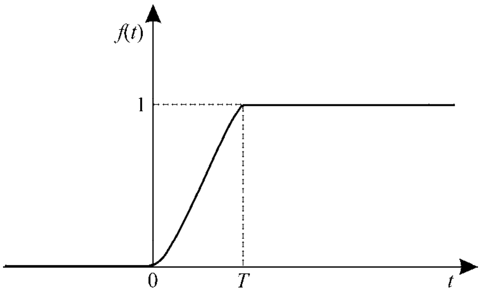

Fig. 1 Schematic micro-scale level kinetics of the fracture of a sample at the break point

In other words, the incubation time introduced above is equal to the duration of the pre-fracture process after the stress in the material has reached the static strength on the given scale level. This duration can be measured in experiments on quasi-static fracture of samples measuring the time of the decrease of stress at the unloading wave front, which can determined by interferometric (visar-based or photoelasticity-based) method using the velocity profile of points on the sample. We analyze examples of the actual utilization of criterion (1) in various physicomechanical problems.

2 Fracture of solids

A typical example illustrating the complicated behavior of the dynamic strength of solids is the time dependence of strength observed under scabbing conditions [5] (Fig. 2). This dependence of the fracture time t_* on the critical pulse amplitude P_* for different pulse durations shows that the dynamic strength is not a material constant but is depending on the time-to-fracture (i.e. sample “lifetime”). The criterion of critical stress $\sigma(t)\leqslant\sigma_c$, where σ_c is the static strength, is able to predict long-term quasistatic fracture caused by long-duration wave pulses $\sigma(t) = P\varphi(t)$, where P is the amplitude and $\varphi(t)$ is the load time profile function. However, in the case of short-duration pulses, fracture time weakly depends on the threshold pulse amplitude, and this dependence has a certain asymptote. This effect is called the phenomenon of the mechanical branch of the strength time dependence. Neither the conventional theory of strength nor known time criteria explains this phenomenon.

Time dependence of strength can be obtained on the basis of the incubation time criterion (1). For the scabbing fracture under consideration, this criterion takes the form of the limiting condition previously proposed in [1-4]:

$$\frac{1}{\tau}\int_{t-\tau}^{t} \sigma\,(t')\,\mathrm{d}t' \leqslant \sigma_c\,, \tag{2}$$

where $\sigma(t)$ is the time dependence of the local stress at the break point. The scheme for the application of criterion (2) to split problems is given by Petrov and Morozov [1,3-4]. An example of a calculation utilizing criterion (2) in order to predict time dependence of the strength of aluminum ($\tau = 0.75\ \mu\mathrm{s}, \sigma_c = 103\ \mathrm{MPa}$) for triangular pulses realized in the experiments reported by Zlatin et al [5] is represented in Fig. 2 by solid line.

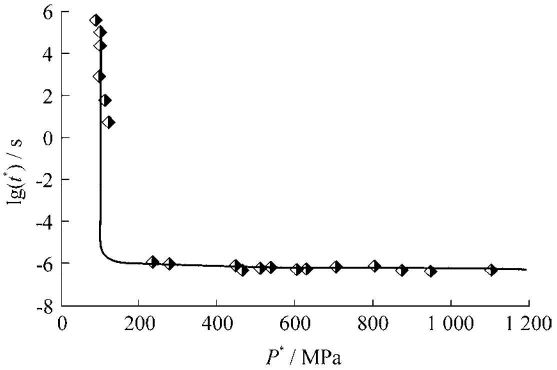

Fig. 2　Logarithm of the fracture-process duration t^* vs. the threshold amplitude P^* of a stress pulse that causes scabbing in an aluminum sample (Zlatin et al [5]).

Incubation time fracture criterion, originally proposed to predict crack initiation in dynamic conditions, was formulated in [1-4]. This criterion for fracture at a point x, at time t, reads:

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x} \sigma(x',t')\,\mathrm{d}x'\mathrm{d}t' \geqslant \sigma_c\,, \tag{3}$$

where τ is the incubation time of a fracture process (or fracture microstructural time): a parameter characterizing the response of the material to applied dynamic loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse or its amplitude). d is the characteristic size of a fracture process zone and is constant for the given material and chosen spatial scale. σ is normal stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress evaluated in quasistatic conditions). x_* and t_* are local coordinate and time.

Assuming

$$d = (2/\pi)(K_{Ic}^2/\sigma_c^2)\,, \tag{4}$$

where K_{Ic} is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasistatic experimental conditions, it can be shown that within the framework of linear fracture mechanics for case of fracture initiation in the tip of an existing mode I loaded crack, Eq. (3) is equivalent to:

$$\frac{1}{\tau}\int_{t-\tau}^{t} K_I(t')\,\mathrm{d}t' \geqslant K_{IC} \tag{5}$$

Condition (2) arises from the requirement that (1) is equivalent to Irwin's criterion $K_I(t) \geqslant K_{Ic}$ in quasistatic conditions ($t \to \infty$). This means that a certain size typical for fractured material appears. This size should be associated with a size of a failure cell on the current spatial scale-all rupture sized essentially less than d cannot be called fracture on the current scale level.

Thus, by introduction of τ and d time-spatial domain is discretized. Once material and scale one is working on are chosen, τ gives a time, such, that energy, accumulated during this time can be released by rupture of the cell that accumulated it. Linear size d assigns dimensions for the cell. Introduction of temporal and spatial domain discretization is a very important step. To our belief, a correct description of high loading rate effects is not possible if this time-spatial discreteness is not accounted somehow. Advantage of incubation time approach is that one can stay within the framework of continuum linear elasticity, utilizing all the consequent advantages and accounting discreetness of the problem only inside critical fracture condition.

As it was shown in multiple publications [4], criterion (3) can be successfully used to predict fracture initiation in brittle solids. For slow loading rates and, hence, times to fracture that are essentially bigger than τ, condition (3) for crack initiation gives the same predictions as classical Irwin's criterion of the critical stress intensity factor. For high loading rates and times to fracture comparable with τ all the variety of effects experimentally observed in dynamical experiments [6-8] can be received using (3) both qualitatively and quantitatively [2-3]. Application of condition (3) to description of real experiments or usage of (3) as a critical fracture condition in finite element numerical analysis gives a possibility for better understanding of fracture dynamics's nature [9-10] and even prediction of new effects typical for dynamical processes [11].

Rate dependences K_{Id} of the dynamic fracture toughness, which were observed in experiments, are characterized by a strong instability and can noticeably change when varying the duration of the load rise stage, the shape of the time profile of a loading pulse, sample geometry, and the method of load application [6-8]. The calculations based on the concept of the incubation time corresponding to the conditions of a number of experiments were carried out by Petrov and Morozov [2]. The results show that the dynamic fracture toughness is not an intrinsic characteristic of a material. Therefore, the employment of both the criterion of the critical intensity coefficient $K_I(t) \leqslant K_{Id}$ and the characteristic K_{Id} as a material parameter defining the dynamic fracture (in analogy to the static parameter K_{Ic}) are incorrect.

3 Interconnection of rupture processes on different scale levels

Consider fracture process that is determined and controlled by the incubation time criterion. Assume that the set of material parameters of the criterion σ_c, d, τ (or σ_c, K_{Ic}, τ) is associated with the given scale level. We suppose that any given scale level is characterized by two characteristic lengths representing its lower and upper limits:

Lower boundary:

$$d \cong (2/\pi)(K_{Ic}^2/\sigma_c^2)\,. \qquad (6)$$

Objects sized essentially less then the lower boundary cannot display fracture on the given scale level. Samples sized essentially less that this boundary cannot be used in order to experimentally evaluate fracture parameters on the given scale level.

Upper boundary:

$$D \cong c\tau\,, \qquad (7)$$

where c is the speed of the energy transport. Thus, biggest possible volume, the energy able to produce fracture can occupy within the time equal to incubation time is introduced. Objects sized essentially more than the upper boundary cann't correctly display fracture parameters on the given scale level. Samples sized essentially more that boundary cann't be used in order to evaluate fracture parameters on the given scale level.

Permissible characteristic sizes L of tested specimens for the given scale level (object sizes for which any fracture theory based on strength properties (σ_c, K_{Ic}, τ) measured on this scale level is valid) should stay within approximate range: $d \leqslant L \leqslant D$.

Thereby it is supposed that any *i*-th scale level is characterized by a pair of linear sizes $\{d_i, D_i\}$ and the range of permissible object dimensions:

$$d_i \leqslant L \leqslant D_i\,. \qquad (8)$$

At the same time we suppose that

$$d_{i+1} = D_i\,, \qquad (9)$$

and therefore

$$\tau_i = d_{i+1}/c\,. \qquad (10)$$

Thus, knowing strength properties (including fracture incubation time) on some scale level it is possible to estimate the upper boundary for smaller scale level, the lower boundary for the larger scale level and the incubation time for the smaller scale level. Modified incubation time fracture criterion is giving a possibility to establish interconnections between fracture on different scale levels. It also provides a possibility to assign fracture scale level for different experiments on the same material.

The abovementioned ideas were applied while modeling propagation of dynamic crack in trunk gas pipeline [12]. In these experiments a section of gas pipe was loaded by internal pressure close to operational pressure inside the gas pipeline. A furrow was made in the part of the pipe parallel to its central axis. The furrow was filled with an explosive substance. When the explosive is blasted the crack starts to propagate from the furrow. Pipelines made of several different pipe steels were tested.

The main difficulty was connected with a fact that the experimental data about strength properties of steel were evaluated on laboratory scale level (samples sized several centimeters) while gas pipeline with diameter of 1.5 m is a steel shell and almost 10 m wide and infinite in another direction was to be modeled. Laboratory tests of relatively small specimens of that particular pipe steel showed large plastic zones and big ductility while large scale tests of the pipes discovered long distant crack propagations that mostly could be interpreted as quasi-brittle behavior on the large scale level. FEM model predicting propagation of crack in the pipe-line was developed. Utilization of material properties received on laboratory scale as input parameters for the FEM model resulted in crack propagation histories drastically differing from the ones observed in reality. The abovementioned approach based on the criterion introduced above was applied in order to modify material properties.

The model was created for a section of pipeline with length of 9 m. Pipeline diameter is 1.22 m [12-13]. Considering large scale fracture of a trunk pipe we used quasi-brittle approach based on the assumption that the size of an element on the crack path is equal to $D = c_1\tau$, where c_1 is the speed of the longitudinal wave in steel (pipeline material) and τ is the fracture incubation time for steel measured on laboratory scale were the fracture process was accompanied by large plastic zone.

FEM modeling was performed for three different steels (X80, X90, X100) [14]. Fig. 3 represents crack extension histories for pipelines made from three different steels and gives an overview of the propagating crack.

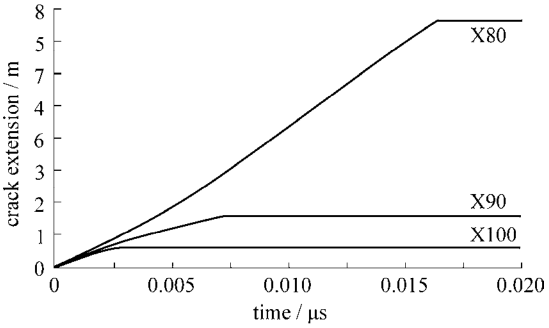

Fig. 3　Crack extension histories for pipelines made of different steels

It was found that in the modeled situation the speed of the crack is close to the speed of the acoustic wave in gas that determines the speed of the front of the pressure drop. This leads to a conclusion about instability of crack propagation regimes in the modeled situation that a small change in properties of the pipeline material can result in qualitative change it crack propagation regime, and should the speed of the crack be higher than the speed of acoustic signal in gas, the crack will never arrest. It was found that length of the resulting crack does strongly depend on material of the pipeline and the length of the resulting crack does vary significantly (from 3 to 300 m) though all steels had very similar properties. The origin of this instability was understood due to numerical analysis presented above. As a result, received crack extension histories were very close to the ones measured experimentally [12-13].

4　Conclusions

Thus, the examples considered above show the fundamental importance of the incubation processes preparing abrupt structural changes (fracture and crack propagation) in continuum under intense pulsed loads. The fracture incubation time is evidently a universal basic characteristic of the dynamic strength and must become one of the main material parameters to be experimentally determined (measured). The above results show that the incubation-time approach is fundamental and makes it possible to adequately represent the dynamics of fracture in solids on various scale levels. A corresponding generalized model accounting for fracture scale level is also presented. It was shown that this can give a possibility to predict fracture on a higher (real) scale level having experimental data obtained on a lower (laboratory) scale. This possibility is of extreme importance for many applications where the possibility to evaluate material strength properties on real structure scale level does not exist (ex. geological objects, big concrete structures, trunk pipelines, etc.)

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固体材料脆性断裂的多尺度时空模型

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摘要：通过实例分析了动态断裂过程所固有的典型动态作用，基于断裂孵化时间理论的时空结构方法提出了固体材料断裂的统一解释. 此外，还提出了计及尺度效应的广义断裂模型，该模型可以用于预测准脆性非均质材料的多尺度断裂，并证明此模型基于实验室尺度的实验数据可以预测更高尺度(真实尺度)的宏观断裂.

关键词：孵化时间准则；动态断裂；动态强度；断裂韧性；时空离散化；多尺度；裂纹扩展；管道；有限元建模

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