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Evaluation of Energy Saving Operational Modes for Industrial Fracture Connected Processes on the Basis of Incubation Time Fracture Criterion

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Abstract: A problem for a central crack in a plate subjected to plane strain conditions is investigated. Mode Ⅰ crack loading is created by a dynamic pressure pulse applied at a large distance from the crack. It was found that for a certain combination of amplitude and duration of the pulse applied, the energy transmitted to the sample has a strongly marked minimum, meaning that with the pulse amplitude or duration moving away from the optimal values, minimum energy required for initiation of crack growth increases rapidly. The results obtained indicate a possibility to optimise energy consumption of different industrial processes connected with fracture. Much could be gained in, for example, drilling or rock pounding where energy input accounts for the largest part of the process cost. Presumably further investigation of the effect observed can make it possible to predict optimal energy saving parameters, i.e. frequency and amplitude of impacts, for industrial devices, e.g. bores, grinding machines, and hence significantly reduce the process cost. The prediction can be given based on the parameters of the media fractured (material parameters, prevalent crack length and orientation, etc.).
Keywords: fracture; energy saving; incubation time fracture criterion

A possibility to optimise the amount of energy, required to fracture materials is of a large interest in connection with many applications. Energy inputs for fracture induced by short impulse loadings are of major importance in such areas as percussive, explosive, hydraulic, electro-impulse and other means of mining, drilling, pounding. In these cases energy input usually accounts for the largest part of the process cost (see Ref.[1]). Taking into consideration the fact that the efficiency of the mentioned processes rarely exceeds a few percent the importance of energy inputs optimization gets evident.

The purpose of the present investigation is to find and explore the amount of energy sufficient to initiate the propagation of a mode I loaded central crack in a plate subjected to plane strain deformation. Two ways to apply the dynamic load to the body are studied. In the first case the load is applied at infinity. The study involves the analysis of interaction of the wave package approaching from infinity with an existing central crack in a plane. The existing crack is oriented parallel to the front of the wave package. In the second case the load is applied at the crack faces. Tractions are normal to the crack faces.

Following the superposition principle these two loading cases should produce identical stress-strain field in the vicinity of the crack tip. It will be shown later that the amount of total energy applied to the body that needed to initiate crack growth is depending on the load application manner in different ways for the two cases under investigation.

1 Load applied at infinity

Fig.1 shows an infinite plane with a central crack. The load is given by the falling wave on the crack. Displacements of the plane are described by:

$$\rho u_{i,tt} = (\lambda + \mu) u_{j,ji} + \mu u_{i,jj} \qquad (1)$$

where "," refers to the partial derivative with respect to time and spatial coordinates; ρ is the mass density, and the indices i and j assume the values 1 and 2. Displace-

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ments are given by u in the direction x; λ and μ are Lame constants. Stresses and strains are coupled by Hooke's law:

$$\sigma_{ij} = \lambda\delta_{ij}u_{k,k} + \mu(u_{i,j} + u_{j,i}) \tag{2}$$

where σ_{ij} represents stresses in direction ij; δ_{ij} is the Kronecker delta assuming value of 1 for $i=j$ and 0 otherwise. Boundary conditions are

$$\sigma_{22}\,|_{|x_1|<l, x_2=0} = \sigma_{21}\,|_{|x_1|<l, x_2=0} = 0 \tag{3}$$

The impact is delivered to the crack by the falling wave:

$$\sigma_{22}\,|_{t<0} = P\Big(H\big(t+\frac{x_2}{c_1}\big) + H\big(t-\frac{x_2}{c_1}\big) - H\big(t+\frac{x_2}{c_1}-T\big) - H\big(t-\frac{x_2}{c_1}-T\big)\Big) \tag{4}$$

where c_1 is the longitudinal wave speed; H is the Heaviside step function and T is the impact duration. P represents the pressure pulse amplitude, Pa. The described problem is solved using finite element method.

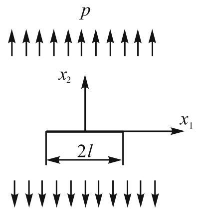

Fig.1 Experiment scheme(Central crack in an infinite plane is loaded by a wave approaching from infinity. Wave front is parallel to the crack plane)

2 Modeling interaction of the wave coming from infinity with the crack

The process is analyzed utilizing the finite element method. ABAQUS[2] finite element package was used to solve the problem. The task was formulated for a quarter sample using the symmetry of the problem about x and y axes. Plane strain conditions were supposed. Area adjacent to the crack tip was meshed with triangular isoparametric quarter-point elements available in ABAQUS package. Thus, mesh in the vicinity of the crack tip may assume a square root singularity in stress/strain fields. About 30×10^5 elements were used to model the cracked sample. Crack surface was represented by 50 nodes along the crack′s half-length. Explicit time integration was utilized to solve the dynamical problem in question.

Computations were performed for granite(E is the elasticity modulus, E = 96.5 Gpa; ρ =2 810 kg/m^3; ν is the Poisson′s ratio, ν =0.29). The results of investigation will qualitatively hold for a big variety of quasi-brittle materials.

In conditions of the plane strain, interaction of the wave approaching from infinity with a central crack was investigated.

Firstly infinite impulse durations were supposed, i.e. $T=\infty$. Time dependence of the stress intensity factor K_I was studied. K_I used in a further analysis was calculated from J-integral that is available as a direct output from ABAQUS solution. Computations were performed for different amplitudes of the loading pulse applied. Typical dependence of K_I on time is presented in Fig.2.

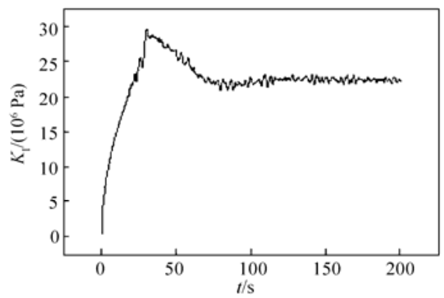

Fig.2 Time dependence typical stress intensity factor in FE solution

Apparently K_I is rapidly approaching the static level. Thus the time to approach the steady-state situation in a vicinity of a crack tip can be estimated as 5—10 times more than the time required by the wave to travel along the crack′s half-length.

Fracture criterion fulfillment was checked for different load amplitudes and durations. Dependence of time-to-fracture T on the amplitude of the load applied was investigated. Time-to-fracture is the time from the beginning of interaction between the wave package and the crack to the crack initiation. Morozov-Petrov incubation time criterion of fracture[3] was chosen to be used. Similar approach to be used in case of short cracks is given by Petrov and Taraban[4].

3 Incubation time fracture criterion

Incubation time fracture criterion proposed in Refs.[3,5,6] is successfully used to describe fracture initiation in dynamic conditions[3,6—8]. Criterion for fracture at a point x at time t reads:

$$\frac{1}{\tau}\frac{1}{d}\int_{x-d}^{x}\int_{t-\tau}^{t}\sigma(x,t)\,\mathrm{d}x\mathrm{d}t \geqslant \sigma_c \tag{5}$$

where τ is the fracture process incubation time (or microstructural time)-parameter characterizing response of a studied material on applied dynamical loads (i.e. τ is constant for a given material and does not depend on problem geometry, the way to apply a load, the shape of a load pulse and its amplitude). d has a meaning of characteristic size of a fracture process zone and is constant for the given material and the chosen spatial scale. σ is stress at a point, changing with time and σ_c is its critical value (ultimate stress or critical tensile stress evaluated in quasistatic conditions).

Assuming

$$d = \frac{2}{\pi}\frac{K_{IC}^2}{\sigma_c^2} \tag{6}$$

where K_{IC} is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasi-static experimental conditions, it can be shown that within the frames of linear fracture mechanics for case of fracture initiation in a tip of an existing crack, loaded by mode I, criterion (5) is equivalent to

$$\frac{1}{\tau}\int_{t-\tau}^{t} K_I(t^*)\mathrm{d}t^* = K_{IC} \tag{7}$$

where x^* and t^* are local coordinate and time.

Condition (6) arises from a requirement that criterion (5) is equivalent to Irwin's criterion ($K_I \geqslant K_{IC}$) in quasi-static conditions ($t \to \infty$). This means that a certain size typical for fractured material appears. This size is believed to be associated with a size of a failure cell on the current spatial scale—all rupture sized significantly less than d cannot be called fracture on the current scale level.

Thus, by introducing τ and d, time-spatial domain is discretized. Once material and scale are chosen, τ gives time, such that energy, accumulated during this time, can be released by rupture of the cell that accumulated it. d assigns dimensions for such a cell. Introduction of time and spatial domain discretization is very important. To our belief, a correct description of high loading rate effects is not possible if this time-spatial discreteness is not accounted some way. Advantage of incubation time approach is that one can stay within the frames of continuum linear elasticity, utilizing all the consequent advantages and accounting discreetness of the problem only in the critical fracture condition.

As it was shown in multiple publications[9—11], criterion (5) can be successfully used to predict fracture initiation at brittle solids. For slow loading rates and, hence, times-to-fracture that are much bigger than τ, condition (7) for crack initiation gives the same predictions as Irwin's criterion[12] of the critical stress intensity factor. For high loading rates and times to fracture comparable with τ, all the variety of effects experimentally observed in dynamical experiments[13—15] can be received using condition (7) both qualitatively and quantitatively[16] Application of condition (7) to the description of real experiments or usage of condition (7) as a critical fracture condition in finite element numerical analysis gives a possibility for better understanding the fracture dynamic nature[7] and even prediction of new effects typical for dynamical processes[8].

Another known approach to dynamic fracture, originating from works of Freund[17] and later developed by Freund[18] and Rosakis[19] is based on an assumption that fracture toughness can be directly and unequivocally coupled with loading rate or stress intensity factor rate. Sometimes, for specific experimental conditions with stress intensity factor (or just stress) monotonously growing with time, such a dependency can be observed in reality. But, generally speaking, the majority of known experimental results for high loading rate fracture stand for inapplicability of this approach. In numerous experiments[20,21] it is observed, that fracture can initiate at a moment when stress intensity factor (or stress, if concerning fracture of intact material, for example, in dynamic cleavage experiments) is decreasing and, hence, is having negative rate. Obviously these phenomena are impossible to describe presuming unequivocal dependency of fracture toughness (or critical stress) on stress intensity factor rate (or stress rate).

All this provides a ground to state that incubation time based approach to fracture has the most potential of all currently known approaches in dynamic fracture.

Using criterion (7) dependence of time-to-fracture on the amplitude of the load pulse applied was studied. Values of $K_{IC} = 2.4\ \mathrm{MPa\cdot m^{\frac{1}{2}}}$ and $\tau = 72\ \mu\mathrm{s}$ typical for granite under investigation were used. Integration of the temporary dependence of stress intensity factor was done numerically. In Fig.3, x-axis represents the time from the beginning of interaction of the wave coming from infinity with the crack to the fracture initiation; y-axis represents the corresponding amplitude of the load applied at infinity. Point in Fig.3 marked with a cross corresponds to the maximum possible time-to-fracture for a given problem. As follows, for the investigated granite and studied experimental conditions fracture is only possible for time less than $92\ \mu\mathrm{s}$.

At the same time the critical (threshold) amplitude of the applied load is found. This amplitude corresponds to the maximum time-to-fracture possible. Loads with amplitudes less than the critical one do not increase the crack's length.

At this point we examine the specific momentum transferred to the plane under investigation by a loading device. In our case

$$P(t) = P(H(t) - H(t-T)) \tag{8}$$

so the specific (per unit of length) momentum of the impact will be

$$R = PT \tag{9}$$

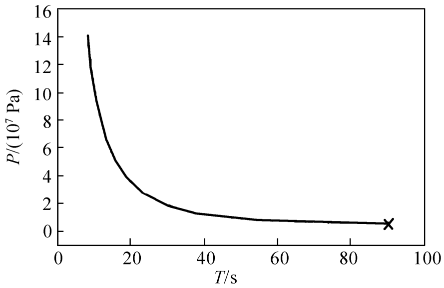

Fig.3 Curve limiting the pulses leading to crack propagation. Time-to-fracture *vs* applied pressure amplitude

The area filled in Figs.4—5 corresponds to a set of momentum values causing fracture. For the values out of this area crack propagation does not occur. The minimum value for the momentum increasing the crack length (44.7 kg • (m/s)) is reached at impulse with duration of 72 μs while the amplitude of the load exceeds the minimal one by more than 10%.

Now we come to examination of the energy transmitted to the sample by a virtual loading device in the process of impact. The shape of the load applied is given by Eq.(8). A specific (per unit of length) energy transmitted to the stripe can be calculated using solution for the uniformly distributed load acting on a half plane. This problem can be easily solved utilising D'Lambet method. Solution for a specific energy transmitted to the half plane appears to be:

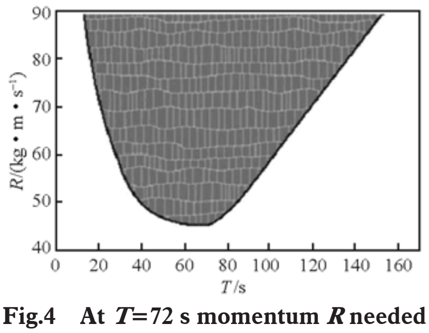

Fig.4 At *T*=72 s momentum *R* needed to advance the crack is minimized

$$\varepsilon_{\mathrm{spec}} = \frac{1}{cp}\int_0^T P^2(t)\mathrm{d}t \tag{10}$$

here c is equal to c_1 and gives the longitudinal wave speed. This result can be used for the problem under investigation as interaction of the loading device and the sample is finished before the waves reflected from the crack come back. Substitution of Eq.(8) into Eq.(10) gives $\varepsilon_{\mathrm{spec}} = \frac{P^2 T}{cp}$.

Analogously to Fig.4, we plot a limiting curve for a set of energies that, being transmitted to the sample, cause the crack propagation (Fig.5)

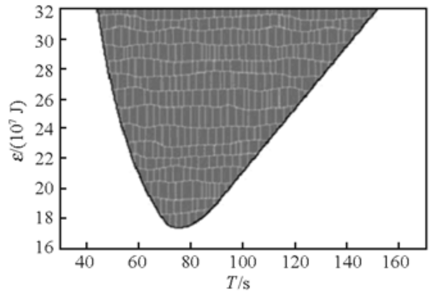

Fig.5 At *T*=78 s energy *ε* needed to advance the crack is minimized

Minimum energy able to increase the crack length (172×10^6 J) is reached at load pulses with duration of 78 μs. As it is evident from Fig.5, minimal energy, required to propagate the crack by impacts with durations differing much from the optimal one, significantly exceeds the minimal possible value. Thus, minimum energy, increasing the crack for the load with duration of 92 μs (at this impact duration crack propagation is possible with the impact of threshold amplitude), will exceed minimal energy possibly by 10%, and at duration of 40 μs it will be more than two times bigger.

4 Case of a load applied at the crack faces

Now we consider a problem similar to the previous one, but with the load applied not at infinity but on the crack faces. The problem is solved numerically in the same manner as the one for the load applied at infinity. Obviously, according to the superposition principle, the solution will coincide with the one for the stripe stretched by a load applied at infinity. Thus all the consequences of the previous solution are applicable, except for the estimations of energy. Specific momentum transmitted to the sample will be the same as the one in the previous problem.

It is not possible to estimate energy transmitted to the sample analytically for the situation, when the load is applied at the crack faces. However the finite element solution can be used in this case to estimate this energy. Fig. 6 represents time dependence of full, kinetic and potential energies of deformation contained in a loaded sample for a particular pressure amplitude.

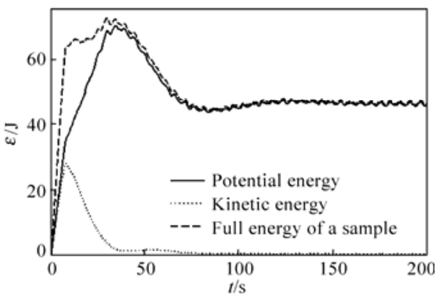

Fig.6 Time dependence of transmitted energy

Firstly the kinetic energy is growing linearly along with the potential one, in the same manner as it happens in the case with the loaded half-plane. However at the moment of time equal to the time sufficient for a wave to travel along the crack length, kinetic energy is starting to be transformed into potential energy of deformation. Some of the energy is returned to the loading device.

Limiting curve for the set of energies increasing the crack length is presented in Fig.7(a). As can be noticed in the case of the load applied at the crack faces, the energy input to increase the crack length has no marked minimum. Minimum energy needed to produce fracture in this case is decreasing with the growth of impulse duration. When the duration is equal to maximal time-to-fracture possible, energy reaches the minimal value.

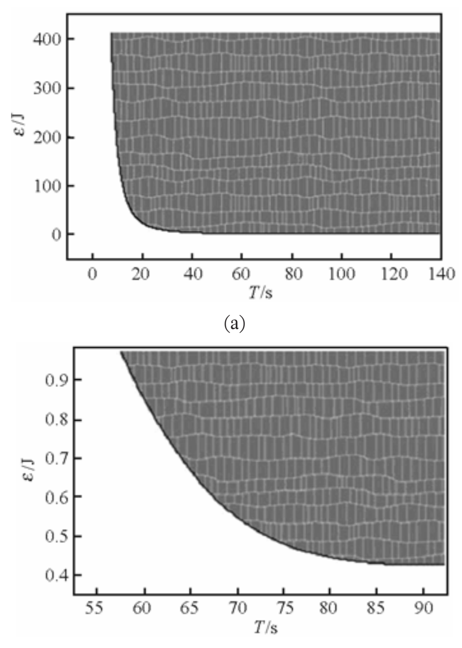

Fig.7 Energy minimization. Possible energy quantities transmitted to a sample by a loading device depending on load duration. Fig.7(b) enlarges part of Fig.7(a)

5 Optimization of the load parameters to minimize energy cost for the crack growth

With the majority of non explosive methods used to fracture materials (drilling, grinding, etc) it is possible to control amplitude and frequency of impacts from the side of a rupture machine. The performed modeling shows that at a certain load duration (at impact fracture of big volumes of material impulse duration is connected to the frequency of the machine impacts) energy inputs for crack propagation has a marked minimum.

Analogously to Fig.5 it is possible to plot the limiting curve for the set of energy values leading to propagation of a crack in the sample at different load amplitudes. This is done in Fig. 8. Thus, it is possible to establish ranges of amplitudes and frequencies of load, at which energy costs for fracture of the material are minimized. These ranges are dependent on parameters of fractured material, predominant length of existing material cracks and the way the load is applied.

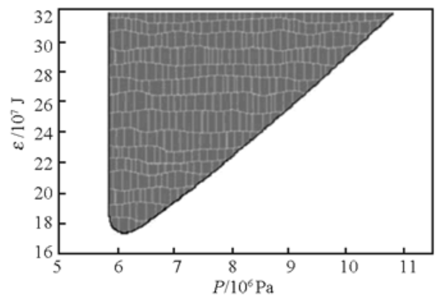

Fig.8 Finding optimal pulse amplitude. Possible energy values for different pressure amplitudes *P*

6 Dependence of the load parameters minimizing the energy for fracture on the length of the existing crack

Dependence of the optimal load parameters on the crack length was also studied. The results obtain are represented in Fig.9(a) and Fig.9(b). As follows from Fig.9(a) duration of the load, that minimizes energy, and momentum inputs are linearly or quasi-linearly dependent on the existing crack length. With the disappearing crack length the duration of the load minimizing momentum needed to increase the crack approaches zero. At the same time the duration optimal for the energy inputs most probably tends to the micro-structural time of the fracture process. Maximum possible time-to-fracture also tends to the micro-structural time of fracture.

Thus, considering intact media as the extreme case of media with cracks when the crack length goes to zero, we find that the maximum possible time-to-fracture is the same as the micro-structural time of the fracture process. Durations of the loads being optimal for the energy inputs for the fracture of intact media are also equal to the microstructure time of the fracture process. Amplitudes of loads, that minimize energy and momentum sufficient to increase the crack length, are presented on Fig.9(b).

As expected, the amplitude of the threshold impulse is inversely dependent on $\sqrt{l}$, where l is the crack length. Dependence of amplitude, minimizing energy inputs, from the crack length is close to $1/\sqrt{l}$. The amplitude, minimizing momentum, is back proportional to the crack length. When the crack length is close to zero, the amplitude of the load, that minimizes the energy cost of the crack propagation, is close to threshold amplitude. However, the amplitude, minimizing the energy input, deviates from the threshold amplitude more and more with the growing crack length (Fig. 10).

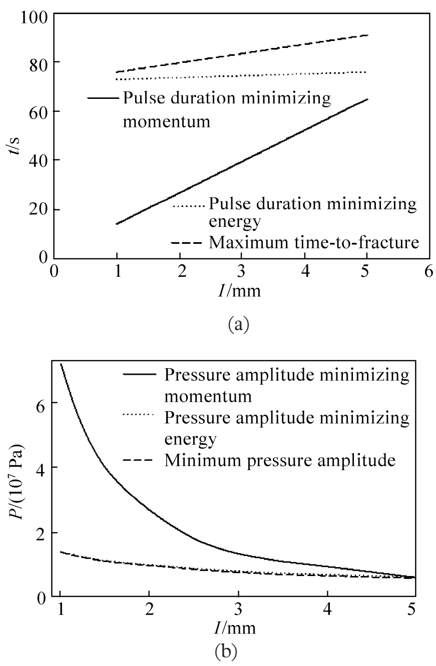

Fig.9 Dependence of optimal load duration and amplitude on crack length

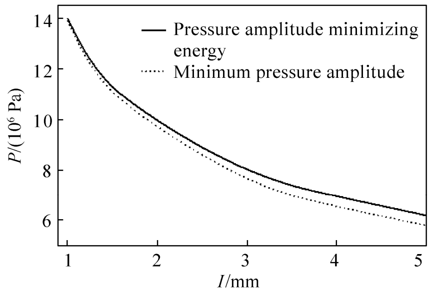

Fig.10 Dependence of optimal load amplitude on crack length

7 Conclusions

The results obtained represent a possibility to optimize energy consumption of different fracture connected industrial processes (drilling, grinding, pounding, etc.). It is shown that the energy cost of crack propagation strongly depends on the amplitude and frequency of the load applied. For example, in the studied problem when the frequency of the load differs from the optimal one by 10%, the energy cost of the crack initiation is exceeding the minimal value by more than 10%.

The obtained dependencies of the optimal characteristics of a load pulse on the existing crack length can help predicting energy saving parameters for the fracture processes investigating the predominant crack size in a fractured material.

Planned research includes a study of the energy costs for fracture of media weakened by a system of cracks of a uniform length. This problem models fracture of media with a predominant size of cracks or defects. It is also important to study energy inputs for fracture of a media with two or more predominant crack lengths.

References

[1] Royal Dutch Petroleum Company Annual Report 2003 [R]. 2003.

[2] *ABAQUS User Manual* [M]. Version 6.4, Hibbit, Karlson & Sorensen, Inc.

[3] Morozov N, Petrov Y. *Dynamics of Fracture* [M]. Springer-Velrag, Berlin-Hidelberg-New York , 2000.

[4] Petrov Y, Taraban V. On double-criterion models of the fracture of brittle materials [J]. *St.-Petersburg University Mechanics Bulletin*, 1997,2:78-81.

[5] Petrov Yu V, Utkin A A. On the rate dependences of dynamic fracture toughness [J]. *Soviet Material Science*, 1989,25(2):153-156.

[6] Petrov Y V, Morozov N F. On the modeling of fracture of brittle solids [J]. *ASME J Appl Mech*, 1994, 61: 710-712.

[7] Bratov V, Gruzdkov A, Krivosheev S *et al*. Energy balance in the crack growth initiation under pulsed-load conditions [J]. *Doklady Physics*, 2004, 49(5): 338-341.

[8] Bratov V, Petrov Y. Optimizing energy input for fracture by analysis of the energy required to initiate dynamic mode I crack growth [J]. *International Journal of Solids and Structures*, 2007, 44: 2371-2380.

[9] Petrov Yu V. On "quantum" nature of dynamic fracture of brittle solids [J]. *Dokl Akad Nauk USSR*, 1991, 321(1): 66-68. [Sov. Phys. Dokl. 36, 802 (1991)]

[10] Petrov Y V, Morozov N F, Smirnov V I. Structural macromechanics approach in dynamics of fracture [J]. *Fatigue Fract Engng Mater Struct*, 2003,26:363-372.

[11] Petrov Y, Sitnikova E. Temperature dependence of spall strength and the effect of anomalous melting temperatures in shock-wave loading [J]. *Technical Physics*, 2005,50: 1034-1037.

[12] Irwin G. Analysis of stresses and strains near the end of a crack traversing a plate [J]. *Journal of Applied Mechanics*, 1957,24: 361-364.

[13] Ravi-Chandar K, Knauss W G. An experimental investigation into dynamic fracture [J]. *International Journal of Fracture*,1984,25: 247-262.

[14] Dally J W, Barker D B. Dynamic measurements of initiation toughness at high loading rates [J]. *Experimental Mechanics*, 1988,28:298-303.

[15] Smith G C. An Experimental Investigation of the Fracture of a Brittle Material [D]. California Institute of Technology,1975.

[16] Petrov Y V. Incubation time criterion and the pulsed strength of continua: Fracture, cavitation, and electrical breakdown [J]. *Doklady Physics*, 2004,49:246-249.

[17] Freund L B, Clifton R. On the uniqueness of plane elastodynamic solutions for running cracks [J]. *Journal of Elasticity*, 1974,4:293-299.

[18] Freund L B. *Dynamic Fracture Mechanics* [M]. Cambridge Press, Cambridge,1990.

[19] Owen D M, Zhuang S, Rosakis A J *et al*. Experimental determination of dynamic crack initiation and propagation fracture toughness in aluminum sheets [J]. *International Journal of Fracture*, 1998,90:153-174.

[20] Shockey D A, Erlich D C, Kalthoff J F *et al*. Short pulse fracture mechanics [J]. *Journal of Engineering Fracture Mechanics*,1986, 23: 311-319.

[21] Zlatin N A, Pugachev G S. Temporal dependency of metal strength [J]. *Solid State Physics*, 1975, 17:2599-2602.

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