

## Minimization of Fracture-Pulse Energy under Contact Interaction

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The pulse fracture of continua is met in various fields of industry, where the processing of materials is carried out in the dynamic mode. As examples of such manufacturing, we can mention the vibration processing of metals, well boring, and drilling or crushing of rocks. The most energy-intensive is the last form of manufacture, and knowledge of the action modes optimal from the viewpoint of the energy consumption could substantially increase the productivity. It is necessary to note that the traditional approaches used in calculating and designing the processing devices based on the principles of dynamic attack disregard the time parameters of the fracture process itself. Usually, only the classical strength criterion, which describes well the fracture process under quasi-static action [1, 2], is used in the calculations and fails to explain a number of effects that are observed, for example, when processing metals and in the case of an additional ultrasonic action on the cutter. For certain values of vibration frequencies, it is possible to reduce strongly the cutter clamping force [3]. For example, under increasing vibration frequency, the fracture intensity (metal removal) increases; however, there is an optimum frequency after which, when exceeded, this value decreases [4]. Because the vibration frequency is inversely proportional to the duration of a single action, it is possible to assume that there is an optimum duration of loading at which the energy necessary for the fracture-pulse generation is minimal. In other words, there is an optimum time of supplying the energy into the fracture zone providing the peak efficiency of the action.

### DETERMINATION OF THRESHOLD AMPLITUDES AT SET FRACTURE-PULSE DURATIONS

When solving the dynamic problems of fracture, the criteria obtained by the extrapolation of the quasi-static criteria of fracture into the dynamic region are frequently used; in this case, the inertial characteristics of material and the duration of action [5] are taken into account, but the time parameters of the fracture mechanism itself are disregarded. In these cases, it is impossible to obtain an estimate of critical characteristics valid for both short and quasi-static actions. In this work, we use the incubation-time criterion for determining the parameters of fracture pulses. This approach is based on the concept of the fracture characteristic time, the time parameter being taken into account in this concept as the strength characteristic of material (the incubation time), which allows one to solve efficiently the problems on the prediction of dynamic strength of continua [6]. The use of this approach is justified by the fact that, for obtaining the above effects, it is necessary to consider the pulse durations in a wide range of values.

In a number of the simplest cases, the criterion for determining the threshold fracture amplitude has the following form [7]:

$$\frac{1}{\tau} \int_{t-\tau}^t \frac{\sigma(s)}{\sigma_s} ds \leq 1, \quad (1)$$

where  $\sigma_s$  is the static ultimate strength (the yield strength of the material) and  $\tau$  is the fracture incubation time. We assume that the pulse shape is approximated by a smooth function

$$\omega(t) = \begin{cases} \exp \frac{1}{1 - \left( \frac{2t}{t_0} - 1 \right)^{-2}}, & \left| t - \frac{t_0}{2} \right| \leq t_0, \\ 0, & \left| t - \frac{t_0}{2} \right| > t_0, \end{cases}$$

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where  $t_0$  is the pulse duration. Then the pulse action can be set as

$$\sigma(t) = \sigma_{\max} \omega(t), \quad (2)$$

where  $\sigma_{\max}$  is the maximum stress. After substituting Eq. (2) into criterion (1), the threshold amplitudes are equal to those values of  $\sigma_{\max}$  for which the following equality is achieved in Eq. (1):

$$\sigma^* = \frac{\sigma_s \tau}{\max_{t \in [0; t_0]} \int_{t-\tau}^t \omega(s) ds}.$$

### SOLUTION OF THE MODELING PROBLEM

As an example of the method of energy delivery to the fracture region, we consider the problem about the contact impact interaction. In [8] on the basis of the Hertz classical scheme, the maximum stress in the medium and the duration of the action arising as a result of the impact of a spherical particle of radius  $R$  having velocity  $V$  on an elastic half-space are calculated. According to the Hertz hypothesis, the expression for the contact force  $P$  has the following form:

$$P(t) = kh^{3/2},$$

$$k = \frac{4}{3}\sqrt{R} \frac{R}{1-v^2}, \quad (3)$$

where  $h$  is the approach,  $E$  is the Young modulus, and  $v$  is the Poisson ratio of the elastic medium. The maximum approach  $h_0$  is found from the formula

$$h_0 = \left( \frac{5mV^2}{4k} \right)^{2/5}, \quad (4)$$

where  $m$  is the particle mass. The total impact duration is

$$t_0 = \frac{2h_0}{V} \int_0^1 \frac{d\gamma}{1-\gamma^{5/2}} = 2.94 \frac{h_0}{V}. \quad (5)$$

The approach  $h(t)$  as a function of time is approximated with high accuracy by the expression

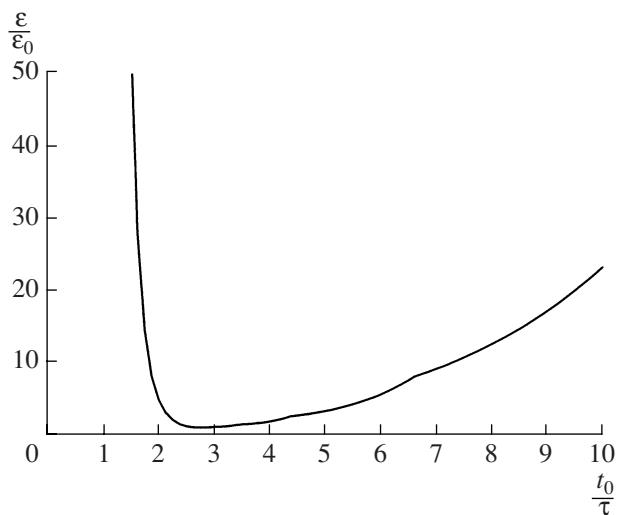
$$h(t) = h_0 \sin \frac{\pi t}{t_0}. \quad (6)$$

The dependence of the maximum rupture stress on time is calculated from the formula

$$\sigma(V, R, t) = \frac{1-2v}{2} \frac{P(t)}{\pi a^2(t)}, \quad (7)$$

where the contact-area radius  $a(t)$  is determined as

$$a(t) = \left( 3P(t)(1-v^2) \frac{R}{4E} \right)^{1/3}. \quad (8)$$



Dependence of the fracture-pulse energy on the impact duration.

Knowing the pulse amplitude and duration, we determine the particle mass and velocity from (3)–(8):

$$R = \frac{t_0}{2.94} \left( \frac{6}{5} \frac{\sigma_{\max}}{\rho(1-2v)} \right)^{1/2},$$

$$V = \left( \frac{5\rho\pi(1-v^2)}{4E} \right)^2 \left( \frac{6}{5} \frac{\sigma_{\max}}{\rho(1-2v)} \right)^{5/2},$$

where  $\rho$  is the particle density and  $\sigma_{\max}$  is the maximum stress, i.e., the pulse amplitude. After the calculation of the particle kinetic energy, we obtain the estimate of the energy necessary for fracture,

$$\epsilon = \alpha \frac{t_0^3 \sigma_{\max}^{13/2}}{\rho^{3/2} E^4},$$

where  $\alpha = \frac{2}{3} \frac{\pi^5}{(2.94)^3} \left( \frac{5(1-v^2)}{4} \right)^4 \left( \frac{6}{5(1-2v)} \right)^{13/2}$  is the dimensionless factor. We plot the graph for the energy in the dimensionless coordinates (figure), where we use the characteristics of the alloy Inconel 718 ( $E = 204.9 \times 10^9 \text{ N/m}^2$ ,  $v = 0.284$ ,  $\sigma_s = 0.7 \times 10^9 \text{ N/m}^2$ , and  $\tau = 8 \mu\text{s}$ ) as the material parameters for which a significant decrease in the rupture strength was experimentally observed in [3] at the pulsed energy delivery in the ultrasonic range of durations.

On the plot in the figure, it can be seen that the lowest of energy is achieved at the load durations of about  $3\tau$ . Thus, knowing the fracture incubation time for the material, we can select the most energetically favorable mode of treatment. In particular, adjusting the duration of impacts, we can optimize the operation of rupture devices of the impact type. Similarly, it is possible to choose the vibration modes for decreasing the energy losses during processing of various materials.

Thus, as a result of using the structural-time approach in which the time parameters of the fracture mechanism are taken into account, we succeeded in giving an explanation for the experimentally observed effect of optimization of the energy spent on the fracture of materials by pulsed dynamic attacks. By the example of the simplest problem about contact collision, we estimated the energy necessary for the fracture-pulse generation in the elastic medium. It was found that this energy substantially depends on the attack duration and has the characteristic lowest value.

## REFERENCES

1. L. A. Vašberg, L. P. Zarogatskii, and V. Ya. Turkin, *Vibration Mills: Fundamentals of Calculation, Designing, and Technological Application* (VSEGEI, St. Petersburg, 2004) [in Russian].
2. I. F. Goncharevich, *Vibration—Nonstandard Way: Vibration in Nature and Technology* (Nauka, Moscow, 1986) [in Russian].
3. A. V. Mitrofanov, V. I. Babitsky, and V. V. Silberschmidt, *J. Materials Processing Technol.* **153/154**, 233 (2004).
4. A. P. Babichev, *Vibration Processing of Details* (Mashinostronie, Moscow, 1974) [in Russian].
5. O. P. Barsukov, L. A. Vašberg, B. V. Semkin, and V. A. Tsukerman, *Izv. Vyssh. Uchebn. Zaved., Fiz.*, No. 7, 96 (1972).
6. E. E. Pavlovskaya and Yu. V. Petrov, *Mekh. Tverd. Tela*, No. 4, 39 (2002).
7. Yu. V. Petrov, *Dokl. Akad. Nauk* **395** (5), 621 (2004).
8. Yu. V. Petrov, *Quantum Micromechanics of Dynamical Fracture of Solids* (St. Petersburg, 1996) [in Russian].

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