ISSN 1028-3358, Doklady Physics, 2007, Vol. 52, No. 10, pp. 565–567. © Pleiades Publishing, Ltd., 2007.
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MECHANICS

Application of the Incubation Time Criterion to the Description of Dynamic Crack Propagation

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Received May 7, 2007

PACS numbers: 62.20.Mk

DOI: 10.1134/S1028335807100138

INCUBATION TIME CRITERION

The incubation time criterion describing fracture initiation under dynamic conditions was originally formulated in [1–3]. According to this, a condition of fracture at point x and time t can be written as

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x^*,t^*)dx^*dt^* \ge \sigma_c, \tag{1}$$

where τ is the incubation time of a fracture process, that is, the parameter characterizing the response of the fractured material to the applied dynamic loads (this writing means that τ is constant for a given material, which does not depend on the specimen geometry, the way by which the load is applied, and the shape and amplitude of the load pulse); d has the meaning of the characteristic fracture size and depends on the given material and the scale on which the experiment is conducted; σ is the time-dependent stress at a local point; σ_c is its critical value under static conditions, which is characteristic of the material under study and the chosen scale; and x^* and t^* are the local coordinate and time, respectively.

Assuming that

$$d = \frac{2}{\pi}\frac{K_{IC}^2}{\sigma_c^2}, \tag{2}$$

where K_{IC} is the critical stress intensity factor, it can be shown that, within the framework of the linear fracture mechanics, condition (1) for mode-I-loaded cracks is equivalent to

$$\frac{1}{\tau}\int_{t-\tau}^{t}K_I(t^*)dt^* \ge K_{IC}. \tag{3}$$

Formula (2) is derived from the requirement that criterion (3) repeats the classical critical stress intensity criterion ($K_I \ge K_{IC}$) in the static case ($t \to \infty$).

Numerous studies (see, e.g., [4, 5]) have shown that a quite adequate description of the initiation of dynamically loaded cracks is possible on the basis of criterion (3). It should be noted that, for slowly applied loads and, therefore, fracture times much greater than τ, condition (3) reduces to Irwin's classical criterion. At the same time, the application of criterion (3) to brittle failure processes with the time to failure comparable with τ makes possible both the description of the entire variety of experimentally observed effects of dynamic fracture (see, e.g., [6 – 8]) and the prediction of new phenomena in the fracture dynamics (see, e.g., [9, 10]).

This study presents the very first attempt at applying criterion (1) to description of the crack propagation dynamics.

CLASSICAL EXPERIMENTS OF RAVI-CHANDAR AND KNAUSS

In order to check for the applicability of criterion (1) to description of the crack propagation dynamics, we modeled the conditions of the classical experiments of Ravi-Chandar and Knauss [6], in which a rectangular specimen was loaded on the crack faces by a dynamic pulse uniformly distributed along the crack length. The temporal dependence of pressure pulse in this system is approximated well by two successive trapezoids (Fig. 1).

The behavior of the system is described by the equations of linear elasticity:

$$\rho u_{i,tt} = (\lambda+\mu)u_{j,ji} + \mu u_{i,jj}, \tag{4}$$

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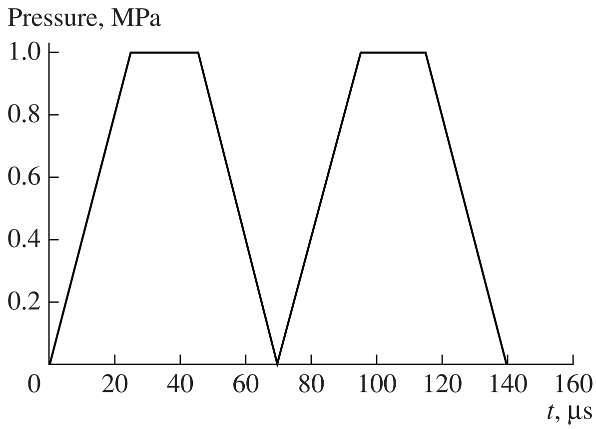

Fig. 1. Temporal variation of the pressure applied to the notch banks.

where the comma denotes the partial derivative with respect to time and spatial coordinates, ρ is the mass density, and the subscripts i and j take the values 1 and 2. The displacements are given by u_i in the directions x_i, respectively, t is the current time, and λ and μ are the Lamé constants. The stresses and strains are related by Hooke's law,

$$\sigma_{ij} = \lambda\delta_{ij}u_{k,k} + \mu(u_{i,j} + u_{j,i}), \quad (5)$$

where σ_{ij} is the stress in the direction ij and δ_{ij} is the Kronecker delta function. At moment $t = 0$, the specimen is free of stresses and the velocities of all points are zero:

$$\sigma_{ij}|_{t=0} = u_{,t}|_{t=0} = 0. \quad (6)$$

The boundary conditions at the crack faces are as follows:

$$\sigma_{21}|_{x_1<0,\, x_2=0} = 0. \quad (7)$$

The load is given by the normal pressure at the crack faces:

$$\sigma_{22}|_{x_1<0,\, x_2=0} = Af(t). \quad (8)$$

where $f(t)$ is the function plotted in Fig. 1 and A is the load pulse amplitude.

Table 1. Properties of homalite-100 material used in the simulations

Density ρ, kg/m^3	1230
Young's modulus E, MPa	3900
Poisson constant ν	0.35
Critical stress intensity factor K_{IC}, MPa $\sqrt{m}$	0.48
Critical tensile stress σ_c, MPa	35
Incubation time of the fracture process τ, ms	9

In order to check the applicability of criterion (1) to description of the crack propagation dynamics, the experimental conditions [6] were modeled using the finite-element method.

FINITE-ELEMENT MODEL

In order to obtain a closed description of the dynamic fracture problem, Eqs. (4) to (8) are supplemented with fracture condition (1). Due to the symmetry of the problem, the crack can propagate only along the x_1 axis. It is assumed that, when criterion (1) is fulfilled, a new surface is formed at a certain point on the x_1 axis.

The problem formulated by Eqs. (1) and (4)–(8) has been solved numerically. Equations (4)–(8) were implemented using the finite-element package ANSYS [11], while the fulfillment of condition (1) was checked using an external program at each time step of the calculations.

Rectangular four-node elements were used to mesh the fractured sample. In the vicinity of the crack continuation, the dimensions of elements were set exactly equal to $d = \frac{2}{\pi}\frac{K_{IC}^2}{\sigma_c^2}$. Taking into account the symmetry across the x_1 axis, we solved the problem only for the upper half of the specimen. The specimen dimensions were the same as those in the simulated experiments. The nodes on the crack continuation were subjected to symmetric boundary conditions until the moment at which condition (1) is fulfilled at the corresponding node. At this moment the restrictions on the displacements of the corresponding node were removed and a new free surface was formed (failure). The properties of the material used in the simulation are given in Table 1.

RESULTS

After the problem formulated above has been solved using the finite-element package ANSYS and the external program controlling the fracture propagation, the data on the K_I time dependence and the crack extension history are provided for further analysis.

Varying the pressure pulse amplitude A, it was found that for amplitudes close to 5 MPa, the crack extension histories were close to those experimentally observed by Ravi-Chandar and Knauss [6]. Figure 2 shows the results for the pulse with an amplitude of $A = 5.1$ MPa in comparison to the experimental data.

Thus, using the finite-element method for solving the dynamic problem of linear elasticity together with criterion (1), it is possible to describe correctly the propagation of dynamically loaded cracks. Criterion (1) with the value of d chosen from the condition of the correspondence of this criterion to the classical Irwin criterion for the fracture under static conditions can be

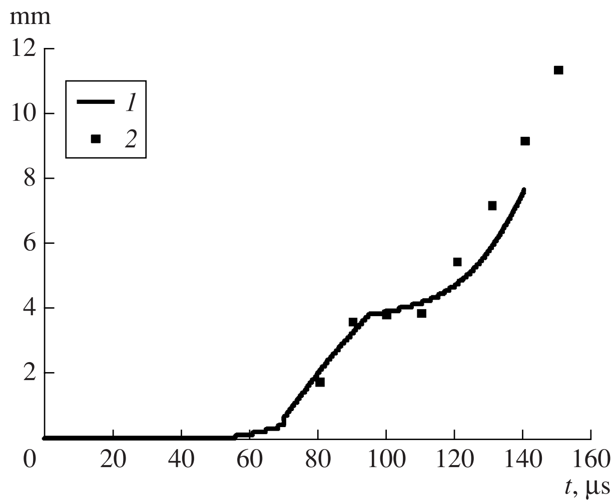

Fig. 2. Crack propagation schedule. Comparison of the calculated and experimental results: (*1*) finite element method; (*2*) experimental points by Ravi-Chandar and Knauss [6].

used to describe the dynamic start, propagation, and arrest of cracks.

Using this approach, it is also possible to describe the propagation of cracks along a trajectory which is not known a priori (e.g., for cracks capable of changing the propagation direction and even of branching). In this case, the fulfillment of condition (1) must be checked for all directions surrounding the crack tip.

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Translated by M. Lebedev