

NUMERICAL INVESTIGATION OF STRESS INTENSITY FACTOR – CRACK VELOCITY RELATION FOR A DYNAMICALLY PROPAGATING CRACK

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Abstract. In this paper incubation time fracture criterion is applied to perform numerical investigation of relation between stress intensity factor (SIF) and crack velocity observed in experiments by J.F. Kalthoff [1] for Araldite B. Two sample geometries were studied – double cantilever beam (DCB) and single edge notched sample (SEN). Quasistatic loading of these samples revealed existence of two branches of stress intensity factor – crack speed dependence corresponding to each sample geometry. Finite element method was used to perform numerical simulations of experiments by J.F. Kalthoff and to study non-unique SIF – crack speed dependence.

1. Introduction

Uniqueness and even existence of stress intensity factor – crack velocity dependence for a moving crack have been studied by many researchers [2, 3]. Classic approach, based on fundamentals of dynamic fracture mechanics, supposes unique stress intensity factor (K) – crack velocity (\dot{a}) dependence which is regarded as a material property [5]. However, studies on dynamic crack growth due to high rate loading [3] revealed that varying stress intensity factor might correspond to a crack with a constant velocity. J.F. Kalthoff [1] conducted experiments on dynamic crack propagation in Araldite B specimens of different shape (double cantilever beam and single edge notched sample) due to quasistatic mode I loading. This work confirms existence of the stress intensity factor – crack speed dependence, however this dependence appeared to be different for samples with different geometry – K values for the DCB sample appear to be up to 20 percent higher than those for the SEN specimen. Geometry dependent $K - \dot{a}$ dependence was also observed in work [5] where Homalite 100 and KTE epoxy specimens of various shapes were investigated.

Such variety of experimental data on the $K - \dot{a}$ dependence exhibits need in a universal approach for the problems of dynamic crack propagation. Classic approaches based on critical stress intensity factor or ultimate stress concepts are not able to adequately predict and explain behavior of a fast moving crack. Application of classic fracture criteria taken from static fracture mechanics to problems of dynamic crack growth leads to discrepancies between experimental data and results of numerical analysis. In order to properly predict fracture in case of dynamically moving crack, transient effects and inertia of the medium should be taken into account and thus appropriate fracture criteria should be used.

The aim of this work is to numerically simulate crack behavior in the DCB and SEN

specimens of Araldite B, observed in work by J.F. Kalthoff [1], using finite element method and incubation time approach to predict movement of the crack.

2. Incubation time fracture criterion

Incubation time criterion [6, 7] for brittle fracture at a point x^* at time t^* , reads as

$$\frac{1}{\tau} \int_{t^*-\tau}^{t^*} \frac{1}{d} \int_{x^*-d}^{x^*} \sigma(x, t) dx dt \geq \sigma_C, \quad (1)$$

where τ is incubation time – a characteristic time of a fracture process which is responsible for reaction of the material to application of dynamic loads, σ_C is ultimate stress of the studied material, $\sigma(x, t)$ is stress at point x and time t . Spatial size d can be calculated using expression $d = 2K_{IC}^2/\pi\sigma_C^2$, where K_{IC} is the critical stress intensity factor. This formula is obtained from the requirement of coincidence of (1) with Irwin-Griffith critical stress intensity factor fracture criterion in case of square root singularity. This parameter should be regarded as a characteristic size of the fracture process zone, being minimal size of the fractured medium for the preset scale level (e.g. minimal crack increment of the crack growth). In condition (1) stress field is supposed to be time-dependent and integration over time indicates that the history of stresses is taken into account or, in other words, the information about processes preceding fracture is controlled by a single measurable parameter – τ . t^* is time when macro fracture occurs which can be calculated from (1) if d, τ and σ_C are given and if stress field $\sigma(x, t)$ is known (either from analytic solution or numeric computations). Within the framework of the incubation time approach, fracture is not an instantaneous event, being result of series of complicated processes preceding fracture (e.g. growth of microcracks or coalescence of micropores). Incubation time parameter makes it possible to take into account these effects representing characteristic time needed for the fracture to develop.

One should note here that for cases of slow application of loading, which results in times to fracture much higher than τ , criterion (1) is reduced to Novogilov-Neuber fracture criterion [8, 9]. Additionally, incubation time criterion is capable of brittle fracture prediction in those cases when stress field is not characterized by square root singularity (e.g. angular notches).

3. Crack propagation experiments and simulations

In [1] authors presented experimental results on dynamic crack propagation in Araldite B specimens of three types: double cantilever beam (DCB), single edge notches specimen (SEN) and a mixed type. The authors utilized method of caustics to measure dynamic stress intensity factor. Position of the crack tip was also known at each moment of time and thus crack velocity values were available in course of the experiment. All the specimens had an artificial initial crack, which started to grow due to quasistatic loading. Shape and dimensions of the investigated specimens are shown in Fig. 1.

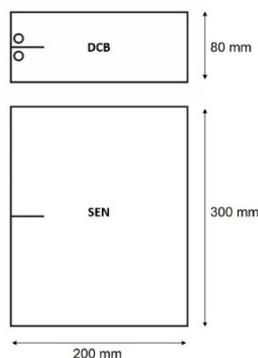


Fig. 1. Geometry of specimens studied in [1].

To simulate experiments, discussed in [1], finite element method was used. FEM software ANSYS with additional C++ routine, which controls crack propagation process were used. The C++ code calculated integral from (1) and compared it to the ultimate stress of the material at every time step of the solution.

In the discussed experiments crack propagated across the sample separating it into two equal parts and thus crack propagation path coincided with the symmetry line for the samples. This feature was used in course of the simulation and only half of the sample was simulated with an appropriate application of symmetry conditions. Material properties for Araldite B were taken from [10]. Incubation time is not known for the studied material. It was chosen to be of one order with PMMA incubation time used in [11] and equal to $1.1 \mu s$.

Crack movement was implemented using node release technique. If relation (1) held in a particular node on the crack path, displacement restrictions were removed from this node and crack tip moved to a subsequent node. One should note here that element size was chosen to be equal d and thus minimal increment of the crack propagation was equal to the structural size used in (1).

Material of the simulated specimens was supposed to exhibit linear elastic behavior. The following initial and boundary value problem was solved

$$\begin{aligned}
 \rho \frac{\partial^2 U_i}{\partial t^2} &= (\lambda + \mu) \nabla_i (\nabla \cdot \vec{U}) + \mu \Delta U_i, \\
 \sigma_{i,j} &= \delta_{i,j} \lambda \nabla \cdot \vec{U} + \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right), \\
 \vec{U}(X, t = 0) &= \frac{\partial \vec{U}}{\partial t}(X, t = 0) = 0, \\
 \sigma_{i,j}(X, t = 0) &= \frac{\partial \sigma_{i,j}}{\partial t}(X, t = 0) = 0, \\
 U_y(X \in \Gamma_1, t) &= vt, \\
 U_y(X \in \Gamma_2, t) &= 0 \text{ – symmetry condition,} \\
 \sigma_y(X \in \Gamma_3, t) &= \sigma_{xy}(X \in \Gamma_2 \cup \Gamma_3, t) = 0.
 \end{aligned} \tag{2}$$

Here $X = (x_1, x_2) = (x, y)$ is the coordinate couple, $\vec{U} = (U_1, U_2) = (U_x, U_y)$ is the displacement vector and V is movement rate of the tensile machine cross head. See Fig. 2 for details.

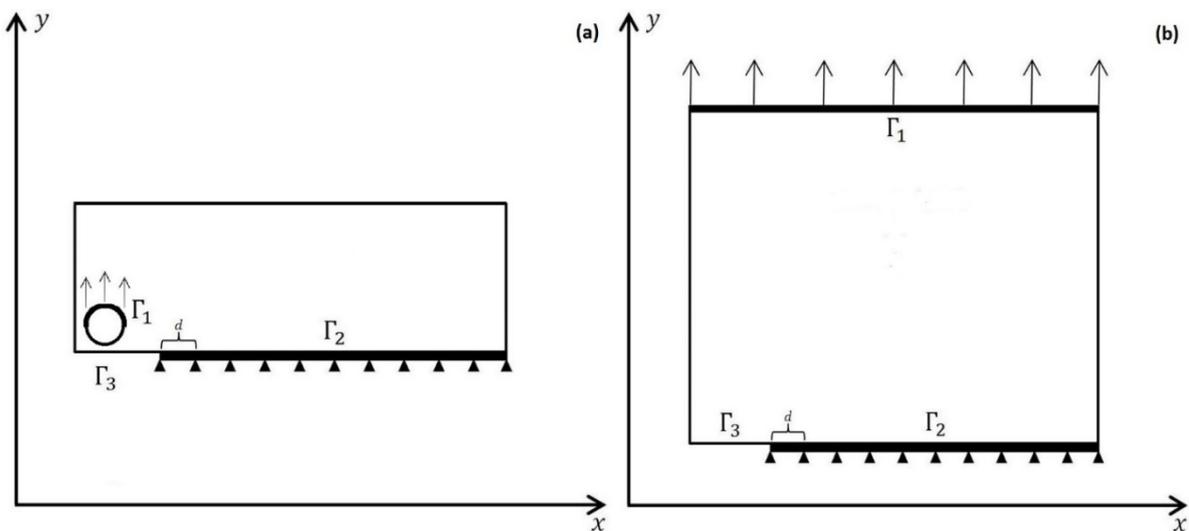


Fig. 2. Simulation scheme for the DCB sample (a) and SEN sample (b).

Time step for the solution was chosen so that the fastest wave could not cross the element during one time step. At each load step of the solution K_I was calculated using J -integral method.

4. Simulation results

Figure 3 demonstrates both experimental and computed $K - \dot{a}$ dependencies for DCB and SEN specimens of Arladite B. The DCB values of the stress intensity factor appear to be around 20 % higher than for the SEN sample, which fits well experimental data.

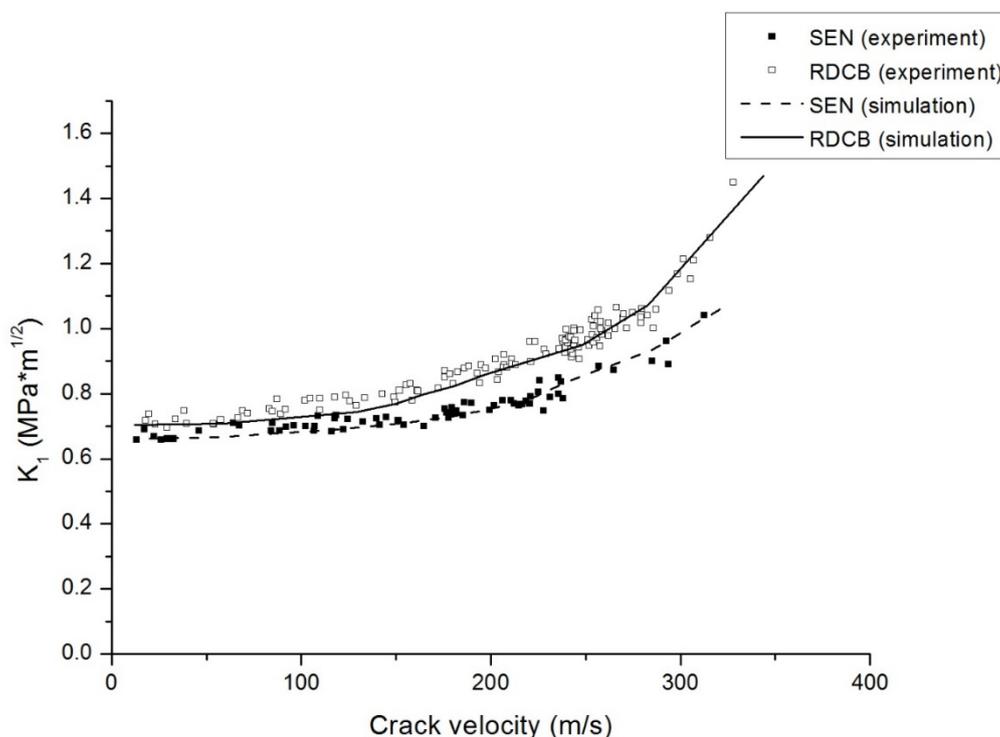


Fig. 3. $K - \dot{a}$ for specimens of different shape. Experimental data and results of the simulation.

Incubation time fracture criterion appears to be a robust tool for simulation of crack movement in various conditions including both quasistatic and dynamic loading [11, 12]. As seen from results of the presented work, effect of multiple branches of stress intensity factor – crack velocity dependence can be also investigated using incubation time approach.

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