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Multi-scale dynamic fracture model for quasi-brittle materials

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ABSTRACT

The dynamic fracture of quasi-brittle heterogeneous materials is governed by processes at several different scale levels. Each of these processes is either independent or dependent on the others. In order to model the dynamic fracture of such materials, it is necessary to account for all the rupture processes that contribute to the overall fracture process. This paper presents a structural-temporal approach for the analysis of the multi-scale nature of dynamic fracture based on the notion of a spatial–temporal fracture cell for different scale levels. The problem of the experimental determination of fracture parameters at a given scale level and their possible interconnections with higher and lower scale levels are discussed. It is shown that these interconnections can permit the prediction of fracture parameters on a higher (real) scale level based on the test data obtained on a lower (laboratory) scale. This predictive capability is of vital importance in many applications in which it is not possible to evaluate the dynamic material properties on the real structural scale level (e.g. geological formations, large concrete structures, trunk pipelines, etc.).

1. Introduction

The universality of the equations of solid mechanics which include a minimal number of material parameters does not permit the description of the diversity of behavior exhibited by various solid materials. An adequate description of the behavior of a given material may require the introduction of additional parameters that account for the structural peculiarities of this material. These additional parameters should however be sufficiently universal, i.e. they should not depend on the experimental conditions; they should not be too numerous and should lend themselves to experimental measurement.

The dynamic fracture of quasi-brittle heterogeneous materials is governed by processes at several different scale levels. Each of these processes is either independent or dependent on the others. In order to model the dynamic fracture of such materials, it is necessary to account for all the rupture processes that contribute to the overall fracture process.

The most common difficulty in predicting the dynamic fracture of typical heterogeneous quasi-brittle materials (concrete, rocks) is the lack of adequate experimental data on the material properties at the relevant scale level. The tests on these materials are performed on laboratory scale samples (ranging in size from several centimeters to a couple of meters), while the size of most structures made of them ranges from tens of meters (concrete structures) to kilometers (geological objects). It is not easy or even possible to obtain reliable experimental data on samples of such large dimensions. It is thus necessary to predict the behavior of the material on the large size level using the properties measured on a smaller scale level. In order to do so certain assumptions have to be made consistent with the knowledge about the dependency of the fracture

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properties on the process scale level. Should these assumptions be confirmed by experiment at the laboratory scale, then it will be possible to predict the fracture properties on the "next" scale level to the laboratory scale. It is even conceivable that for some quasi-brittle materials it will be possible to predict fracture on any scale level from the properties measured on the laboratory scale level.

Unfortunately, no satisfactory approach to the problem exists even today. Several attempts have been made to compare the strength properties of the same material on different scale levels (Petrov, Gruzdkov, & Morozov, 2005), but these have not provided a systematic explanation of the observed dependencies.

In this paper, we shall propose a model for the dynamic fracture of quasi-brittle heterogeneous materials on different scale levels based on our experience in experimental, theoretical and numerical investigations of multi-scale fracture. The idea of interconnected fracture scale levels originates from the concept of a fracture cell implicit in the quasi-static fracture criterion introduced by Neuber and Kerbspannungslehre, (1937) and later, but independently, by Novozhilov (1969a,b) and in its later generalization to dynamic fracture by Morozov and Petrov (2000), Petrov (1991) and Petrov and Morozov (1994). The generalization to dynamic fracture involves the notion of incubation time, thus introducing a spatial–temporal discretization of the fracture process.

The incubation time fracture criterion, originally proposed in Petrov and Morozov (1994), Morozov and Petrov (2000) and Petrov (1991) for predicting crack initiation under dynamic loading conditions, states that fracture will initiate at a point x at time t when

$$\frac{1}{\tau}\int_{t-\tau}^{t}\frac{1}{d}\int_{x-d}^{x}\sigma(x',t')dx'\,dt' \geq \sigma_c. \tag{1}$$

Here, τ is the incubation time of the dynamic fracture process (or the fracture micro-structural time). It characterizes the response of the material to the applied dynamic loads; it is constant for a given material in the sense that it does not depend on the geometry of the test specimen, the way the load is applied, or the shape or amplitude of the load pulse. d is a characteristic size of the fracture process cell (zone) and is a constant for the given material and the chosen spatial scale. σ is the normal stress at the point which varies with time and σ_c is its critical value (i.e. the ultimate tensile strength evaluated under quasi-static conditions).

Assuming, as in the Irwin's small scale yielding approximation, that

$$d = \frac{2}{\pi}\frac{K_{Ic}^2}{\sigma_c^2} \tag{2}$$

where K_{Ic} is the mode I fracture toughness, measured under quasi-static experimental conditions, it can be shown that within the framework of linear elastic fracture mechanics (LEFM), the dynamic crack initiation criterion (1) for an existing mode I loaded crack is equivalent to:

$$\frac{1}{\tau}\int_{t-\tau}^{t}K_I(t')dt' \geq K_{Ic} \tag{3}$$

This follows from the requirement that (1) is equivalent to Irwin's criterion, $K_I(t) \geqslant K_{Ic}$, under quasi-static conditions ($t \to \infty$). This means that a certain size characterizing the fractured material appears in the dynamic fracture initiation criterion. This size is associated with the size of the failure cell on the current spatial scale – all ruptured cells sized less than d cannot be regarded as failure cells on the current scale level.

Thus, by the introduction of τ and d the temporal–spatial domain is discretized. Once the current working scale for a given material has been chosen, τ gives the time in which the energy accumulated in the cell of size d is enough to rupture it. We believe that a correct description of high loading rate effects requires the introduction of this temporal–spatial discreteness. The advantage of the incubation time approach is that one can remain within the framework of continuum linear elasticity and allow for the discreteness of the dynamic fracture process only inside the critical fracture condition.

As has been demonstrated previously (Petrov, 1991; Petrov, Morozov, & Smirnov, 2003; Petrov & Sitnikova, 2005), the dynamic fracture criterion (3) successfully predicts fracture initiation in brittle solids. For slow loading rates when the times to fracture are much longer than τ, the criterion (3) is equivalent to Irwin's criterion (Irwin, 1957). For high loading rates when the times to fracture are comparable with τ, a variety of effects observed in dynamical experiments (Dally & Barker, 1988; Kalthoff, 1986; Ravi-Chandar & Knauss, 1984; Smith, 1975) has been explained qualitatively and quantitatively using (3) (Petrov, 2004). The application of (3) for the description of real experiments or in the finite element analysis of dynamic fracture allows us to gain a better understanding of the nature of dynamic fracture (Bratov, Gruzdkov, Krivosheev, & Petrov, 2004) and even to predict new effects typical for dynamical processes (Bratov & Petrov, 2007a,b).

2. Interconnection of rupture processes on different scale levels

Consider a dynamic fracture process that is described by the incubation time criterion. Assume that the set of material parameters in this criterion σ_c, d, τ (or σ_c, K_{Ic}, τ) has been determined at a given scale level. We now assume that the characteristic length at this scale level is bounded from above and below. Thus, material volumes smaller in size than the lower bound

$$d \cong \frac{2}{\pi}\frac{K_{Ic}^2}{\sigma_c^2} \tag{4}$$

cannot display fracture at this scale level. Therefore, test samples smaller in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level.

The upper bound of the characteristic length is

$$D \cong c\tau \tag{5}$$

where c is the speed of the energy transport in the material. This sets an upper limit on the volume of material in which the total energy accumulated during the incubation period is able to produce fracture. Test samples larger in size than this bound cannot be used to determine experimentally the dynamic fracture parameters at this scale level.

The admissible characteristic size L of the test specimen at a given scale level (at which fracture properties, σ_c, K_{Ic}, τ, are measured) should be in the range:

$$d \leq L \leq D$$

It follows that the i-th scale level is characterized by a pair of linear sizes $\{d_i, D_i\}$ and the admissible range of test specimen size is

$$d_i \leq L \leq D_i. \tag{6}$$

We now choose

$$d_{i+1} = D_i \tag{7}$$

so that

$$\tau_i = \frac{d_{i+1}}{c} = \frac{D_i}{c} \tag{8}$$

Thus, knowing the strength properties and the fracture incubation time on some scale level, it is possible to estimate the upper bound for the smaller scale level, the lower bound for the larger scale level and the incubation time for the smaller scale level. In this manner, the incubation time dynamic fracture criterion allows us to establish interconnections between the fracture properties on different scales. It also makes it possible to assign the fracture scale levels for different experiments on the same material.

3. Modeling propagation of cracks in trunk gas pipelines

The above multi-scale approach was applied to the modeling of dynamic crack propagation in a trunk gas pipeline (Abakumov, 2008; Igi and Akiyama, 2008). The data on the strength properties of the pipeline steel were obtained on laboratory scale level (on test samples several centimeters in size), while the actual pipe had a diameter of 1.22 m and was (practically) infinitely long. Laboratory tests on relatively small notched specimens of the pipe steel exhibited large plastic zones and ductile tearing, while large pipe sections failed in tests after long crack propagation without significant plastic deformation akin to quasi-brittle fracture. The multi-scale approach generally is based on the scaling of the material properties from the laboratory scale to the large scale of the tested pipe sections.

In many practical situations for which nonlinear fracture mechanics is more appropriate another approach based on a generalization of the spatial–temporal approach can be followed to account for substantial plastic deformations (e.g. in highly plastic steels) or large "fracture process zones" (e.g. in quasi-brittle materials like concretes or rocks). A large "process zone" observed at the laboratory scale level can be regarded as a material parameter that remains constant in absolute size as the sample size increases. Thus, the "process zone" for large samples will play the same role as a "small scale yielding zone" of fracture (allowing the application of LEFM) for much larger scale fracture processes. This (sudden) transition from large scale "yielding" to small scale "yielding" can result in catastrophic fracture as, for example, in the case of dynamic fracture of gas pipelines (Abakumov, 2008; Igi and Akiyama, 2008) or the fracture of large concrete beams (Karihaloo, 1995,1999). The analysis of fracture in these situations can be effectively performed using the "nonlocal" alternative of the spatial–temporal approach. In this alternative approach the incubation time criterion can be written as:

$$\int_{t-\tau}^{t}\int_{x-l_p}^{x} \sigma(x', t') f(x') dx' \, dt' \geq G_F \cdot \tau \tag{9}$$

where $f(x) = -\frac{dw}{dx}$ is a weight function describing the spatial rate of the crack opening $w = w(x)$, l_p is the 'process zone' size and G_F is the specific fracture energy, i.e. the energy dissipation per unit area in the 'process zone'. In (Karihaloo, 1995,1999) several different variants of the weight function have been proposed for quasi-brittle materials. Mathematically, the only difference between the criteria (1) and (9) is in the choice of the weight function $f(x)$. In the simplest special case of a linear crack opening in the process zone $f(x) = 1$ so that $G_F = \sigma_c \cdot l_p$, where σ_c is the ultimate strength in quasi-static conditions and τ is the incubation time of fracture measured in laboratory scale tests of the dynamic strength of 'defect-free' standard samples. The dynamic fracture criterion (1) that is a generalization of the classic static LEFM criterion can be recovered as a limiting case of (9) for the linear crack opening function and a small 'process zone'.

The linear crack opening approximation is the easiest for analysis and convenient for practical qualitative estimations, although it is not suitable for analyzing the precise behavior of the fracture zone. It gives results that are qualitatively similar to those of the more sophisticated nonlinear models of large "process zones". Should further research however suggest the need for a nonlinear model, the present analysis will not require major modifications. Assume that tests have been performed on samples which are "sufficiently large" in size on a given scale level in the sense that the size of the "process zone" approaches its upper limit (Karihaloo, 1999): $l_p \to l_{p\infty} = D = c \cdot \tau$ and $G_F \to G_D = \sigma_c \cdot D$. Then, for the prediction of fracture of sufficiently large structures made of nonlinear ("plastic") materials or quasi-brittle materials with extensive "process zones" it is sufficient to use the following fracture condition:

$$\int_{t-\tau}^{t} \int_{x-D}^{x} \sigma(x', t') dx' dt' \geq G_D \cdot \tau \tag{10}$$

The "non-local" spatial–temporal criterion (10) was used to conduct a qualitative analysis of the above problem of crack propagation in a trunk gas pipeline. The large scale fracture of a pipe section (length 9 m, diameter 1.22 m) was modeled. The size of an element on the crack path was chosen equal to $D = c_1\tau = 90$ mm, where c_1 is the speed of the longitudinal wave in the pipe steel and τ is the fracture incubation time measured on laboratory scale specimens of this steel in which the fracture process was accompanied by large plastic deformations.

The pipe was subjected to internal pressure close to the operational pressure in the gas pipeline. The drop in pressure in the pipeline as a result of crack extension was modeled by the motion of two wave fronts: the front wave of the pressure drop (velocity of this front is equal to the velocity of the acoustic wave in the gas – approximately 400 m/s for natural gas) and the back front of the pressure drop (traveling at a lower speed). After the passage of the back front, the pressure inside the pipeline is equal to the external atmospheric pressure. Between the two fronts, the pressure was assumed to vary linearly with the distance along the pipe. The fracture was initiated from a small artificial precursor crack, mimicking the appearance of a real crack in the pipeline.

The incubation time fracture criterion (10) was used to predict the conditions for release of the nodes along the crack path (i.e. for the initiation of crack growth). The modeling was performed on three different grades of steel (X80, X90 and X100), differing only in the ultimate tensile strength (625 MPa for X80, 711 MPa for X90, and 748 MPa for X100). Other material properties of all three grades were the same: density $\rho = 7800$ kg/m^3, Young's modulus = 210 GPa, Poisson's ratio = 0.3, and the fracture incubation time $\tau = 15$ μs. Fig. 1 shows the crack extension histories of the pipeline made from three different steels. Fig. 2 shows a typical propagating crack in the modeled pipe section. The color bar scale shows the magnitude of displacement of points on the pipe.

It was found that the speed of the crack is close to the speed of the acoustic wave in the gas (i.e. the speed of the wave front of the pressure drop). The small difference in the ultimate tensile strength of the pipeline steel leads to a qualitative change in the crack propagation regime; if the speed of the crack exceeds the speed of acoustic wave in the gas, the crack will never arrest.

The instability of crack propagation regimes revealed by the finite element analysis is in good agreement with experimental observations on dynamic cracking in gas pipelines (Abakumov, 2008). In the experiments a section of the pipe (length 9 m) was subjected to an internal pressure close to the operational pressure inside the gas pipeline. A furrow was made in a part of the pipe surface along its length. This furrow was filled with an explosive. When the explosive was detonated, a crack started to propagate from the base of the furrow. Pipe sections made of several different pipe steel grades were

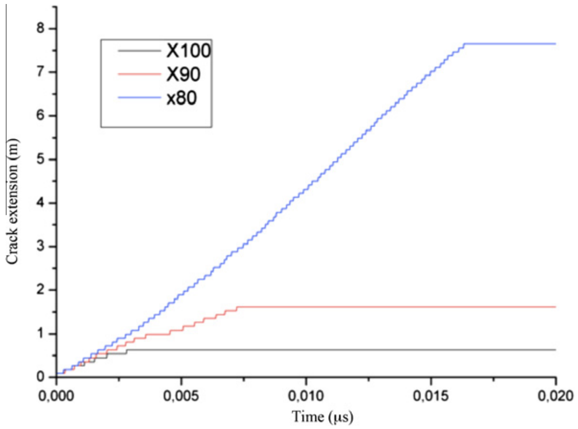

Fig. 1. FEM crack extension histories of a pipeline made of different steels.

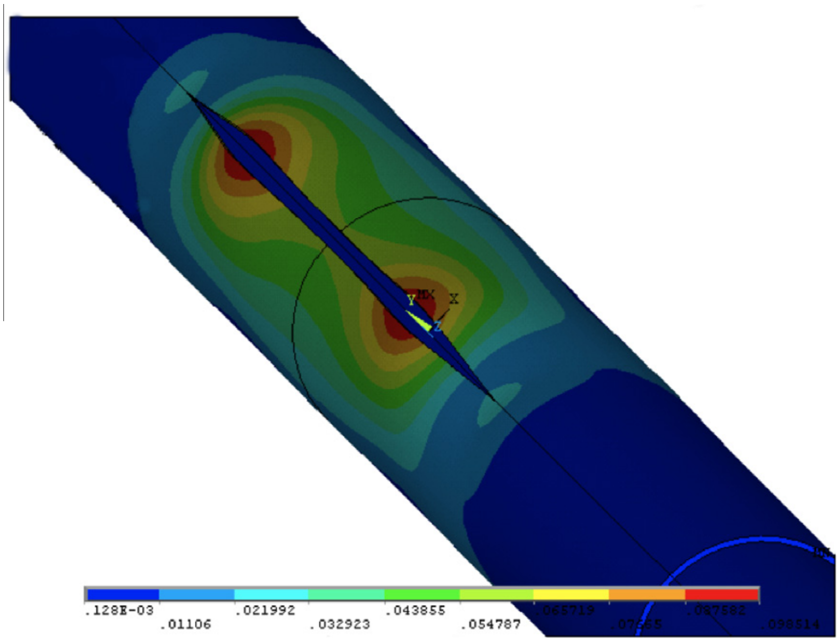

Fig. 2. Pipeline section with propagating crack.

tested. It was found that length of the crack before it arrested depended strongly on the ultimate tensile strength of the pipe steel and varied over two orders of magnitude from 3 to 300 m, despite the fact that the ultimate static tensile strength of the three steel grades differed by less than 20%.

4. Dynamic fracture of CARDIFRC

The high-performance fiber reinforced concrete, CARDIFRC, developed and produced at Cardiff University (Benson & Karihaloo, 2005a,b; Benson, Nicolaides, & Karihaloo, 2005) was tested using the Hopkinson Split Bar (HSB) equipment (with steel bar diameter 20 mm) (Akopov et al., 2003). Standard Brazil tests were conducted on circular discs of CARDIFRC (diameter 15 mm and thickness 10 mm). The standard Kolsky method was used.

These experiments were analyzed using the incubation time theory. The incubation time criterion for dynamic fracture (1) was applied in order to calculate the variation of the critical failure stress as a function of the stress or strain rate. The dynamic split tensile strength can be calculated on the basis of the following critical condition flowing from (1) in the special case that the material is initially 'intact':

$$\frac{1}{\tau}\int_{t-\tau}^{t}\sigma(t')\,dt' \le \sigma_c \tag{11}$$

Here, $\sigma(t)$ is time dependence of the local tensile stress at the fracture point, σ_c is the quasi-static split tensile strength (for CARDIFRC = 23 MPa), and τ is the incubation time of fracture.

The test results and the predictions based on (11) are presented in Fig. 3. The best agreement between the test results and predictions is achieved when the incubation time τ equals 15 μs.

In order to demonstrate the possible effect of loading rate on the strength of a material a comparison is made between CARDIFRC and granite. The dynamic split strength (stress at fracture) of the latter obtained under the same experimental conditions is also shown on Fig. 3 as a function of stress rate. The quasi-static split tensile strength of granite is 19 MPa and its incubation time is 70 μs (Petrov et al., 2005). As seen on Fig. 3, although CARDIFRC has a higher quasi-static split strength than that of granite, its dynamic split strength is lower at high stress rates ($>10^{2.5}$).

The incubation time for CARDIFRC determined above can be used to estimate the upper bound for the experimental scale level D. $D = c\tau = 67.5$ mm, as c = 4500 m/s for CARDIFRC. Following the previous discussion about the interconnection of rapture processes on different scale levels, we can expect that LEFM can be used on samples much larger in size than 67.5 mm to study the fracture of CARDIFRC. In fact, this is confirmed by the theoretical and experimental investigations reported in Karihaloo (1999) and Karihaloo, Abdalla, and Xiao (2006), albeit on concrete without fibers.

If the size of the process zone in an "infinitely" large specimen is identified with D, i.e. $l_{p\infty} = D = c\tau$ then from Fig. 4 (and in agreement with the observations reported in Karihaloo (1999), Karihaloo et al. (2006)) it is clear that LEFM can be applied to concrete structures with characteristic sizes $L \approx 1.5 - 2$ m (i.e. 20–30 times larger than the estimated $D = 67.5$ mm).

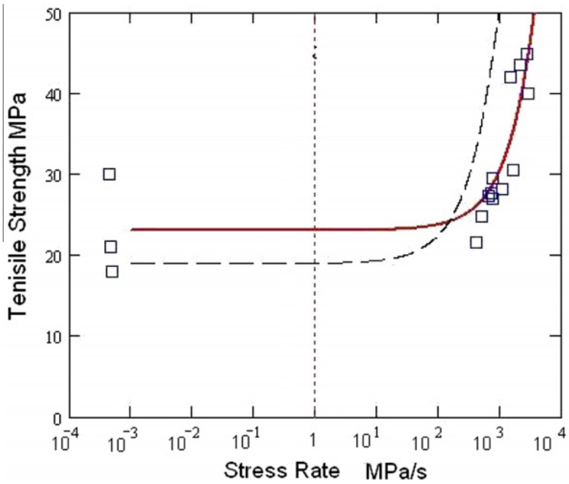

Fig. 3. Dynamic tensile split strength of CARDIFRC measured experimentally and calculated with help of criterion (11) (solid line $\sigma_c = 23$ MPa and $\tau = 15$ μs); dashed line corresponds to granite ($\sigma_c = 19$ MPa and $\tau = 70$ μs-datafrom (Petrov et al., 2005).

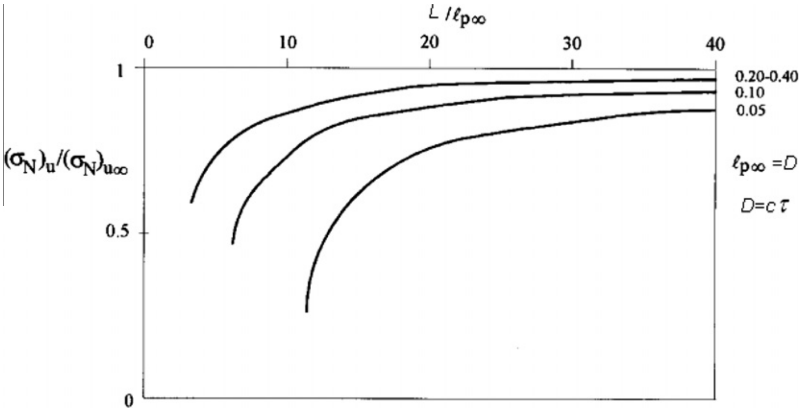

Fig. 4. Variation of the nominal failure strength with depth (Karihaloo, 1999) for several values of the pre-crack size. In the range $0.2 < \alpha < 0.4$ the variation of the nominal failure strength is nearly the same (α = notch to depth ratio).

5. Conclusions

A structural-temporal approach for the analysis of the multi-scale nature of dynamic fracture based on the notion of a spatial–temporal fracture cell for different scale levels was presented. It was shown that the incubation time fracture criterion can be an effective tool to establish interconnections between the dynamic strength properties on different scales.

Two characteristic linear sizes specifying the lower and upper limits on the scale level need to be specified in order to estimate the correct admissible characteristic size of the test specimen at a given scale level. The knowledge of both static and dynamic properties of the fracture process is of principal importance for this specification.

The problem of the experimental determination of the fracture parameters at a given scale level and their possible interconnections with higher and lower scale levels was discussed. It was shown that these interconnections can permit the prediction of the fracture parameters on a higher (real) level based on the test data obtained on a lower (laboratory) level.

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