Doklady Physics, Vol. 49, No. 5, 2004, pp. 338–341. Translated from Doklady Akademii Nauk, Vol. 396, No. 3, 2004, pp. 345–348.

MECHANICS

Energy Balance in the Crack Growth Initiation under Pulsed-Load Conditions

V. A. Bratov, A. A. Gruzdkov, S. I. Krivosheev, and Yu. V. Petrov*

Presented by Academician N.F. Morozov November 17, 2003

Received December 5, 2003

1. ENERGY APPROACH TO THE FRACTURE PROBLEM

The classical approach to fracture mechanics going back to Griffith is based on the statement that a crack propagates if this process leads to a decrease in the total energy ε of a system. For a plate of unit thickness, the crack-growth conditions can be written as

$$-\frac{\partial \varepsilon}{\partial L} = 2\gamma. \quad (1)$$

Griffith initially interpreted the quantity 2γ as the surface energy, because it represented the specific work (per unit area) expended to form a new surface. Irwin and Orowan showed that this quantity should be interpreted as the total work (including the plastic one) in the fracture zone. This work can be taken as the resistance to a certain dissipative process proceeding in a small region near the crack tip. The study of this characteristic includes the determination of its physical origin (different for different classes of materials) and its measurement.

For the fracture near the crack tip loaded by mode I, criterion (1) is equivalent to the criterion for the critical stress-intensity factor $K_I \le K_{Ic}$. For a linearly elastic body, the Griffith constant is equal to

$$\gamma = \frac{K_{Ic}^2}{2E}, \quad (2)$$

where E is Young's modulus. Thus, γ can be indirectly determined in this case from the standard tests for measuring K_{Ic}.

Nowadays, it is generally recognized [1] that the expenditure energy per unit area of a fracture surface essentially depends on the action duration. Other characteristics of dynamic fracture (critical intensity factor, limiting amplitude of applied load) also essentially depend on time. Therefore, the problem of determining the Griffith specific surface energy is urgent for high-rate fracture.

2. DYNAMIC TESTS

It is impossible to adequately study the energy balance by conventional methods of creating short-term loads, because it is very difficult to estimate the energy fraction directly transferred to a specimen. In particular, the parameters of explosive or shock action can be estimated only approximately. For most cases, energy exchange between the specimen and a loading device is rather complicated [2], which makes it impossible to authentically determine the stage at which fracture occurs.

The procedure using a magnetic pulse installation is free of the above disadvantage. Load is formed by the magnetic-pulse method, where the mechanical pressure depends on the spatial configuration of current-carrying elements [3–5]. For a known current distribution, the current-pulse parameters are unambiguously related to magnetic pressure. In addition, the energy state of the specimen at the instant of fracture can be determined quite exactly in many cases. This determination is possible for the following reasons. First, the pressure on cut edges is monitored throughout the pulse (about 1–10 μs). Second, high-speed shooting of the fracture process makes it possible to precisely determine the instant at which cracks start. In certain cases, it can occur after the disappearance (removal) of the external pressure pulse. Third, after the termination of the pulse action, the specimen does not interact with the installation; hence, the specimen under fracture becomes an energetically closed system.

The indicated principles were realized in the run of tests for specimens with a cut modeling a macrocrack. The specimens were manufactured from spheroplastics (120 × 120 × 10-mm specimens with a 60 × 2.2-mm cut) [3] and polymethyl methacrylate (200 × 200 × 10-mm specimens with a 100 × 3-mm cut) [4, 5]. At the cut tips, we made a 0.2-mm-thick notch 3 mm in length.

St. Petersburg State University,
Bibliotechnaya pl. 2, St. Petersburg, 198504 Russia
** e-mail: yp@yp1004.spb.edu*

1028-3358/04/4905-0338$26.00

The cut edges were loaded by uniform pressure approximated by the dependence

$$P(t) = A\sin^2\frac{\pi t}{T}, \quad 0 \le t \le T. \tag{3}$$

For each pulse duration T, the dependence of the crack-growth length on the applied-load amplitude A was studied. We aimed to determine the threshold amplitude. The obtained dependences were well approximated by the linear functions (Fig. 1)

$$\Delta L = k(A - A_0), \quad A > A_0. \tag{4}$$

The quantity A_0 is the threshold amplitude above which the crack begins to grow. This quantity was found by extrapolating the experimental data to the value $\Delta L = 0$.

The high-speed shooting of the fracture process revealed a delay of the onset of crack growth relative to the instant at which the stress-intensity factor reaches its maximum [6]. In this case, the time before the onset of crack growth is appreciably longer than the time of action of the external pressure pulse. Hence, the total energy of the external action is converted to the elastic and kinetic energy of the material, and the crack propagation is further determined by this internal energy. Since the potential of external forces is equal to zero at the instant of fracture near the crack tip, it is possible to consider that the function ε in Eq. (1) coincides with the internal energy of the specimen.

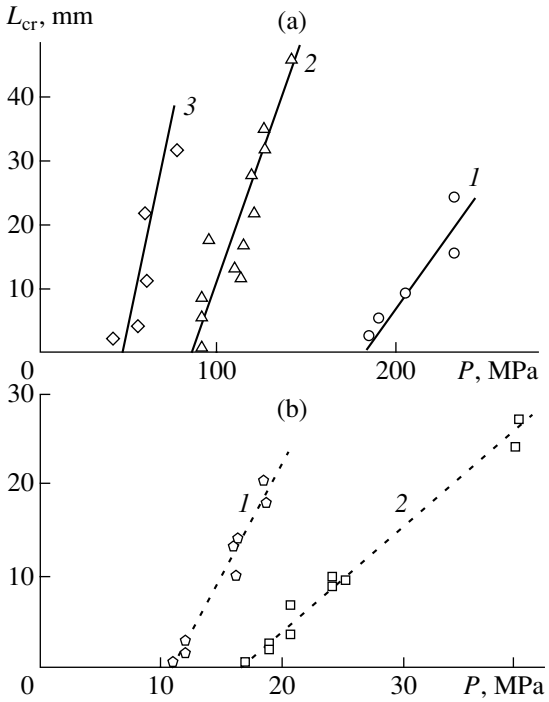

Fig. 1. Experimental data on the length L_{cr} of a crack grown from the cut tip vs. the amplitude P of rupturing pressure pulse for (a) polymethyl-methacrylate specimens at the time of a pulse increase to the maximum (*1*) 1, (*2*) 2, and (*3*) 4.6 μs and for (b) spheroplastic specimens at the time of a pulse increase to the maximum (*1*) 4.4 and (*2*) 2.76 μs.

3. ENERGY BALANCE

We analytically estimate the energy transferred to the specimen due to interaction with the installation. During the load action, the wave has no time to pass along the cut edges; therefore, in the first approximation, it is possible to consider the problem of a plane wave in a half-space:

$$U_{tt} - c^2 U_{xx} = 0,$$

$$U_x(0, t) = -\frac{P(t)}{c^2\rho}, \quad U(x, 0) = 0, \quad U_t(x, 0) = 0.$$

Here, x and U are the coordinate and the longitudinal displacement, respectively; c is the longitudinal-wave velocity; t is the time; and ρ is the density. Solving the problem with Eq. (3), we derive the expression for the energy transferred to the specimen:

$$\varepsilon = \frac{3DHA^2T}{8c\rho}, \tag{5}$$

where D is the cut length, H is the plate thickness, and A and T are the parameters of the pressure pulse from Eq. (3).

To prove the correctness of this approach for estimating the energy transferred to a plate during the contact, we numerically analyzed the three-dimensional problem of the interaction of a specimen with a loading device [7]. Using a finite-element software package, we calculated the energy transmitted to the specimen during the contact with the current-carrying bus. For estimating this energy, it suffices to numerically simulate only an early stage of the process, and precisely during the pressure-pulse action from the bus side, and then to estimate the energy involved in the specimen.

As a result, we obtained data on the total energy of the specimen. Thus, similar to analytical calculation, the total energy of the system including its kinetic component is taken into account.

The simulation was carried out for polymethyl methacrylate specimens at three different durations of the action pulse. The table presents the energies transferred to the specimen as calculated by both Eq. (5) and the finite-element method. The good agreement between them supports the applicability of this

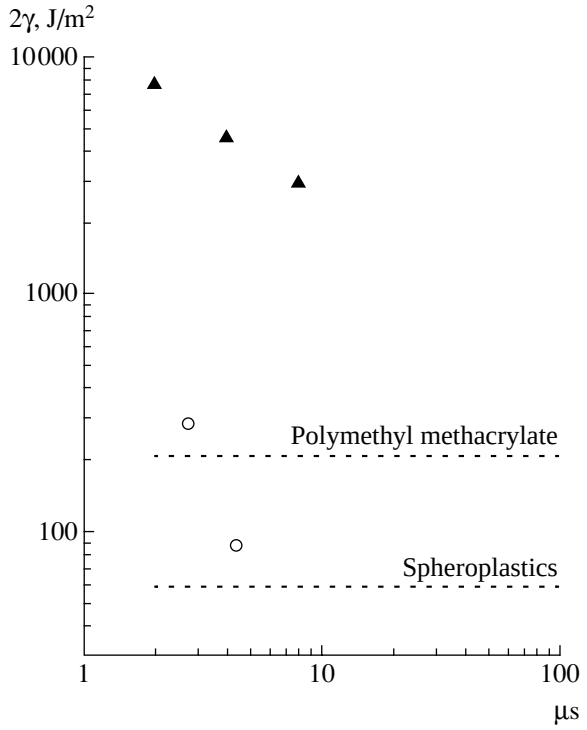

Fig. 2. Fracture energy consumption vs. the loading-pulse duration (the dashed lines correspond to quasistatic tests) for (triangles) polymethyl methacrylate and (circles) spheroplastic.

approach for estimating the energy transferred to the specimen during the contact.

4. CONDITION OF CRACK GROWTH

From Eq. (4), it follows that the crack-length increment is related to the increment of the amplitude of the applied pulsed pressure as $dL = kdA$. Taking into account Eq. (5) and $dS = HdL$, we obtain the specific (per unit area) energy consumption on fracture at the crack onset

$$\left.\frac{d\varepsilon}{dS}\right|_{\Delta L = 0} = \frac{3TA_0D}{4kc\rho}.$$

If the time of travel of an elastic wave along a cut exceeds the time interval from the loading-application instant to the crack onset, i.e., for sufficiently large specimens, only an effective area $D_* = ct_*$, where t_* is the time before the crack onset, instead of the total cut length, must be taken into account when calculating the fracture energy consumption. In this case, we derive the following relation for the specific energy consumption:

$$\frac{3TA_0t_*}{2k\rho} = 2\gamma_d. \quad (6)$$

The quantity γ_d is an analogue of the quantity γ in Eq. (1). Since c and ρ are the known material parameters, and A_0, k, and t_* are determined experimentally for the pulses of various duration T, Eq. (6) yields this quantity as a function of load duration.

Table

T, μs	A, MPa	Energy, J	
		by Eq. (5)	finite-element calculation
2	172	4.17	4.00
4	93	2.44	2.32
8.6	48	1.40	1.33

5. DISCUSSION OF THE RESULTS

Figure 2 shows that γ_d exceeds the value calculated from Eq. (2) using the results of quasistatic tests by more than an order of magnitude. However, as the load duration increases, this value decreases markedly. For polymethyl methacrylate, the points calculated from Eq. (6) lie on a straight line plotted in logarithmic coordinates; i.e., the relation $\gamma_d T^\alpha = \text{const}$ is met for microsecond loads.

The results of the finite-element simulation show that the kinetic and potential components of the energy are comparable. Thus, at the action rates realized in the experiments at the magnetic-pulse installation, it is unjustified to neglect the kinetic energy of a material. Moreover, the inclusion of the kinetic energy can qualitatively change the behavior of the fracture energy consumption.

The basic advantage of the applied testing scheme is that there is no energy exchange between a specimen and a loading device after the termination of the pressure-pulse action, and fracture begins to develop after removing load when the specimen becomes energetically closed.

ACKNOWLEDGMENTS

This work was supported by the Russian Foundation for Basic Research (project nos. 02-01-01035, 03-01-39010, and TsKP 00-01-05020), Swiss Science Foundation, NATO Scientific Affairs Division, and the Russian Federal Program “Integratsiya.”

REFERENCES

1. N. F. Morozov and Yu. V. Petrov, *Dynamics of Fracture* (Izd. S.-Peterburg. Gos. Univ., St. Petersburg, 1997; Springer-Verlag, Berlin, 2000).

2. Yu. V. Petrov and V. I. Smirnov, *Nonstationary Oscillations and Behavior of Internal Energy of One-Dimensional Solids* (S.-Peterb. Gos. Univ., St. Petersburg, 2002).

3. S. A. Atroshenko, S. I. Krivosheev, Yu. V. Petrov, *et al.*, in *Proceedings of the 3rd International Conference "Kharitonov Topical Lectures"* (VNIIÉF, Sarov, 2001), pp. 131–132.

4. S. I. Krivosheev, N. F. Morozov, Yu. V. Petrov, and G. A. Shneerson, Izv. Akad. Nauk, Mekh. Tverd. Tela **165** (5), 78 (1999).

5. S. I. Krivosheev and G. I. Shneerson, in *Strength and Fracture of Materials and Structures: Collection of Scientific Works* (S.-Peterb. Gos. Univ., St. Petersburg, 1999), No. 18, pp. 116–122.

6. A. N. Berezkin, S. I. Krivosheev, Yu. V. Petrov, and A. A. Utkin, Dokl. Akad. Nauk **375**, 328 (2000) [Dokl. Phys. **45**, 617 (2000)].

7. V. A. Bratov and A. A. Gruzdkov, in *Proceedings of the 17th International Conference of Young Scientists (Gagarin Lectures), Moscow, 2001*, p. 119.

Translated by V. Bukhanov