# Multiscale Fracture Model for Quasi-brittle Materials

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**Abstract.** Fracture of quasi-brittle heterogeneous materials is steered by processes at several different scale levels. These processes can progress independently or affect each other. In order to model fracture of such materials one should account for all rupture processes contributing to overall fracture process. This paper is presenting structural-temporal approach for analysis of multiscale nature of brittle fracture. Notion of spatial-temporal cell for different scale levels is introduced. Problem of experimental determination of a fixed scale level is discussed. Possible interconnections of this scale level with higher and lower scale levels are discussed. It is shown that this can give a possibility to predict fracture on a higher (real) scale level having experimental data obtained on a lower (laboratory) scale. This possibility is of extreme importance for many applications where the possibility to evaluate material strength properties on real structure scale level does not exist (ex. geological objects, big concrete structures, trunk pipelines, etc.).

### Introduction

Universality of equations of solid mechanics has an obvious disadvantage – minimal number of parameters is not giving a possibility to describe all the diversity of properties displayed by various materials. For instance, movement of elastic media is defined by ratio of longitudinal and transverse wave speeds. Adequate description of strength requires introduction of parameters accounting for structural peculiarities of fractured materials. Basic requirements to these parameters are: they should be sufficiently universal (i.e. they should not depend on experimental conditions), they should be few and there should exist a possibility to evaluate theses parameters in experiment.

Fracture of quasi-brittle heterogeneous materials is steered by processes at several different scale levels. These processes can progress independently or affect each other. In order to model fracture of such materials one should account for all rupture processes contributing to overall fracture process.

Common difficulty in predicting fracture of typical heterogeneous quasi-brittle materials (concrete, rocks) is the lack of adequate experimental data on material strength on the practical scale level. While most of experiments are performed on laboratory scale (with of samples sized from several centimeters to several meters), in many practical problems fractured objects have sizes from tens of meters (concrete structures) to kilometers (natural objects). It is not easy or even not possible to receive reliable experimental data for samples with such dimensions. At the same time in some cases one can try to predict behavior of material on a bigger size level using strength properties evaluated on a smaller scale level. In order to do so certain assumptions are to be done. These assumptions should be done on the basis of knowledge about dependency of strength on the process scale level. Should these assumptions be correct, this will give a possibility to predict fracture on the scale level "next" to the experimental scale level on which the fracture properties were evaluated. Possibly, for some problems and materials it will give a possibility to predict fracture on any scale level having strength properties measured on a fixed scale level.

Unfortunately to the moment no satisfactory approach to the problem exists. Several attempts to compare strength properties of the same material on different scale level are known [1], but received data are not giving a possibility to systemize the dependencies observed.

In this paper we are trying to sum up our knowledge and experience in experimental, theoretical and numerical investigations of multiscale fracture and apply this to model fracture of quasi-brittle heterogeneous materials on different scale levels. The idea about interconnection of fracture scale levels is originating from concept of fracture cell implicitly introduced by Neuber and Novozhilov in their fracture criterion [2,3,4] and later developed by Petrov and Morozov [5,6]. Currently the Neuber-Novozhilov criterion had evolved into the incubation time fracture criterion introducing spatial-temporal discretization of fracture process.

Incubation time fracture criterion, originally proposed to predict crack initiation in dynamic conditions, was formulated in [5,6]. This criterion for fracture at a point x, at time t, reads as:

$$\frac{1}{\tau} \frac{1}{d} \int_{x-d}^{x} \int_{t-\tau}^{t} \sigma(x^*, t^*) dx^* dt^* \ge \sigma_c .$$

$$\tag{1}$$

where  $\tau$  is the incubation time of a fracture process (or fracture microstructural time) – a parameter characterizing the response of the material to applied dynamic loads (i.e.  $\tau$  is constant for a given material and does not depend on problem geometry, the way a load is applied, the shape of a load pulse or its amplitude). *d* is the characteristic size of a fracture process zone and is constant for the given material and chosen spatial scale.  $\sigma$  is normal stress at a point, changing with time and  $\sigma_c$  is its critical value (ultimate stress or critical tensile stress evaluated in quasi-static conditions).  $x^*$  and  $t^*$  are local coordinate and time.

Assuming

$$d = \frac{2}{\pi} \frac{K_{IC}^2}{\sigma_c^2} \quad . \tag{2}$$

where  $K_{IC}$  is a critical stress intensity factor for mode I loading (mode I fracture toughness), measured in quasistatic experimental conditions, it can be shown that within the framework of linear fracture mechanics for case of fracture initiation in the tip of an existing mode I loaded crack, (1) is equivalent to:

$$\frac{1}{\tau} \int_{t-\tau}^{t} K_{I}(t^{*}) dt^{*} \ge K_{IC}$$
(3)

Condition (2) arises from the requirement that (1) is equivalent to Irwin's criterion ( $K_I \ge K_{IC}$ ), in quasi-static conditions ( $t \rightarrow \infty$ ). This means that a certain size typical for fractured material appears. This size should be associated with a size of a failure cell on the current spatial scale – all rupture sized essentially less than d cannot be called fracture on the current scale level.

Thus, by introduction of  $\tau$  and d time-spatial domain is discretized. Once material and scale one is working on are chosen,  $\tau$  gives a time, such, that energy, accumulated during this time can be released by rupture of the cell that accumulated it. Linear size *d* assigns dimensions for the cell. Introduction of temporal and spatial domain discretization is a very important step. To our belief, a correct description of high loading rate effects is not possible if this time-spatial discreteness is not accounted somehow. Advantage of incubation time approach is that one can stay within the framework of continuum linear elasticity, utilizing all the consequent advantages and accounting discreteness of the problem only inside critical fracture condition.



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As it was shown in multiple publications (ex. [7-9]), criterion (3) can be successfully used to predict fracture initiation in brittle solids. For slow loading rates and, hence, times to fracture that are essentially bigger than  $\tau$ , condition (3) for crack initiation gives the same predictions as Irwin's criterion of the critical stress intensity factor [10]. For high loading rates and times to fracture comparable with  $\tau$  all the variety of effects experimentally observed in dynamical experiments (ex. [11-14]) can be received using (3) both qualitatively and quantitatively [15]. Application of condition (3) to description of real experiments or usage of (3) as a critical fracture condition in finite element numerical analysis gives a possibility for better understanding of fracture dynamics's nature (ex. [16]) and even prediction of new effects typical for dynamical processes (ex. [17,18]).

### Interconnection of rupture processes on different scale levels

Consider fracture process that is determined and controlled by the incubation time criterion. Assume that the set of material parameters of the criterion  $\sigma_c$ , d,  $\tau$  (or  $\sigma_c, K_{Ic}, \tau$ ) is associated with the given scale level.

We suppose that any given scale level is characterized by two characteristic lengths representing its lower and upper limits:

Lower boundary:

$$\boldsymbol{d} \cong \frac{2}{\pi} \frac{\boldsymbol{K}_{lc}^2}{\sigma_c^2}.$$
(4)

Objects sized essentially less then the lower boundary cannot display fracture on the given scale level. Samples sized essentially less that this boundary cannot be used in order to experimentally evaluate fracture parameters on the given scale level.

Upper boundary:

$$D \cong \mathbf{C} \ \tau \tag{5}$$

where c is the speed of the energy transport. Thus, biggest possible volume energy, being able to produce fracture, can occupy within the time equal to incubation time is introduced. Objects sized essentially more than the upper boundary cannot correctly display fracture parameters on the given scale level. Samples sized essentially more that this boundary cannot be used in order to evaluate fracture parameters on the given scale level.

Permissible characteristic sizes L of tested specimens for the given scale level (object sizes for which any fracture theory based on strength properties ( $\sigma_c, K_{Ic}, \tau$ ) measured on this scale level is valid) should stay within approximate range:

$$d \le L \le D$$

Thereby it is supposed that any i-th scale level is characterized by a pair of linear sizes  $\{d_i, D_i\}$  and the range of permissible object dimensions

$$d_i \le L \le D_i \tag{6}$$

At the same time we suppose that

$$d_{i+1} = D_i$$

(7)



and therefore

$$\tau_i = \frac{d_{i+1}}{c} \,. \tag{8}$$

Thus, knowing strength properties (including fracture incubation time) on some scale level it is possible to estimate upper boundary for smaller scale level, lower boundary for larger scale level and incubation time for smaller scale level. Modified incubation time fracture criterion is giving a possibility to establish interconnections between fracture on different scales. It also provides a possibility to assign fracture scale level for different experiments on the same material.

#### Modeling propagation of cracks in trunk gas pipelines

The abovementioned ideas were applied while modeling propagation of dynamic crack in trunk gas pipeline [19]. The main difficulty was connected with a fact that the experimental data about strength properties of steel were evaluated on laboratory scale level (samples sized several centimeters) while gas pipeline with diameter of 1.5 m is a steel shell that is almost 10m wide and infinite in another direction was to be modeled. Laboratory tests of relatively small specimens of that particular pipe steel showed large plastic zones and big ductility while large scale tests of the pipes discovered long distant crack propagations that mostly could be interpreted as quasi-brittle behavior on the large scale level. FEM model predicting propagation of crack in the pipeline was developed. Utilization of material properties received on laboratory scale as input parameters for the FEM model resulted in crack propagation histories drastically differing from the ones observed in reality. The abovementioned approach based on the criterion introduced above was applied in order to modify material properties. As a result, received crack extension histories were very close to the ones measured experimentally [19,20].

The model was created for a section of pipeline with length of 9 meters. Pipeline diameter is 1.22 meters [19]. Considering large scale fracture of a trunk pipe we used quasi-brittle approach based on the assumption that the size of an element on the crack path is equal to  $D = c_1 \tau$ , where  $c_1$  is the speed of the longitudinal wave in steel (pipeline material) and  $\tau$  is the fracture incubation time for steel measured on laboratory scale were the fracture process was accompanied by large plastic zone.

Pipeline was loaded by internal pressure close to operational pressures in gas pipelines. Drop of pressure in the pipeline as a result of crack extension was modeled as movement of two wavefronts: the front of the front wave of pressure drop (velocity of this front is equal to velocity of the acoustic wave in gas – about 400 m/s for natural gas) and the back front of pressure drop travelling at a lower speed. After passage of the back front the pressure inside the pipeline is equal to the external atmospheric pressure. Between the two fronts pressure is supposed to be linearly dependent on coordinate. Fracture was initiated by a small defect (crack) that was artificially introduced. This is imitating appearance of a crack in a pipeline (for example, fatigue crack).

Incubation time fracture criterion is utilized in order to predict conditions for release of the nodes along the crack path.

Modeling was performed for three different steels (X80, X90 and X100). Figure 1 presents crack extension histories for pipelines made from three different steels. Figure 2 gives an overview of the propagating crack.



Figure 1. Crack extension histories for pipelines made of different steels



Figure 2. Pipeline with propagating crack



It was found that in the modeled situation the speed of the crack is close to the speed of the acoustic wave in gas that determines the speed of the front of the pressure drop. This leads to a conclusion about instability of crack propagation regimes in the modeled situation – a small change in properties of the pipeline material can result in qualitative change it crack propagation regime: should the speed of the crack be higher than the speed of acoustic signal in gas, the crack will never arrest.

Received instability of crack propagation regimes is in a good coincidence with experiments on dynamic cracking in gas pipelines [19]. In these experiments a section of gas pipe was loaded by internal pressure close to operational pressure inside the gas pipeline. A furrow was made in the part of the pipe parallel to its central axis. The furrow was filled with an explosive substance. When the explosive is blasted the crack starts to propagate from the furrow. Pipelines made of several different pipe steels were tested. It was found that length of the resulting crack does strongly depend on material of the pipeline and the length of the resulting crack does vary significantly (from 3 to 300 meters) though all steels had very similar properties. The origin of this instability was understood due to numerical analysis presented above.

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