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Analysis of energy required for initiation of inclined crack under uniaxial compression and mixed loading

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ABSTRACT

The central aim of this work is to investigate energy input required for initiation of fracture within quasibrittle media with existing defects. As a model problem, a solution for a central crack in a plane subjected to load applied at infinity is considered. Density of energy that should be introduced into the plane in order to create conditions leading to fracture in the tip of the existing central crack is studied. Situations corresponding to fracture created by purely compressive loading, fracture created by pure shear and fracture created by combination of compressive and shearing load with different intensity ratios are investigated. It is found, that combination of compressive and shearing load is resulting in significantly lower energies spent for creation of fracture as compared to energies corresponding to purely compressive loading. These estimates are qualitatively coinciding with phenomena observed experimentally for processes connected with grinding and fragmentation of quasibrittle heterogeneous media with existing defects (cracks). Earlier it was shown that for grinding of rocks and other similar materials, addition of shear component to compressive impact loading applied to the fractured media results in significant reduction of the process energy consumption. The results received in this paper can be used for prediction of optimal energy saving parameters for industrial machines working for grinding and fragmentation of quasibrittle materials.

1. Introduction

It was observed experimentally (see, ex. [16]) that a combination of compression and shear loading for industrial processes connected with fracture and fragmentation of rock materials results in a significant reduction of process energy consumption as compared to only compressive loading applied to the fractured media.

Initiation and propagation of single and multiple inclined cracks under uniaxial and biaxial loading was studied in numerous publications (see ex. [3,4,5,2] or [6,9]). In these works authors usually investigate applicability of different criteria predicting conditions of fracture initiation and direction of crack propagation under mixed-mode (compression and shear) loading (see ex. [1]). In various works these approaches were implemented into numerical computational schemes predicting propagation of mixed-mode loaded cracks for various geometries and load application modes (ex. [7] or [8]). It is also possible to simulate fracture and deformation on macroscopic scale utilising molecular dynamics (see ex. [23,22]). In spite of extensive amount of results in the area, in the majority of the known publications energy required for initiation or propagation of those cracks is not studied. At the same time, energy that should be spent for fracture is the characteristics having central importance for practical applications related to grinding and fracture of materials (see ex. [13], where dynamic problem was solved). The amount of this energy is particularly important for industrial processes of rock handling and enrichment. These processes are working for fragmentation and grinding of significant volumes of rock material, which is associated with substantial energy consumption (electricity or other energy carrier). In such processes, the cost of energy usually accounts for the bulk of the process cost (see, ex. [18]).

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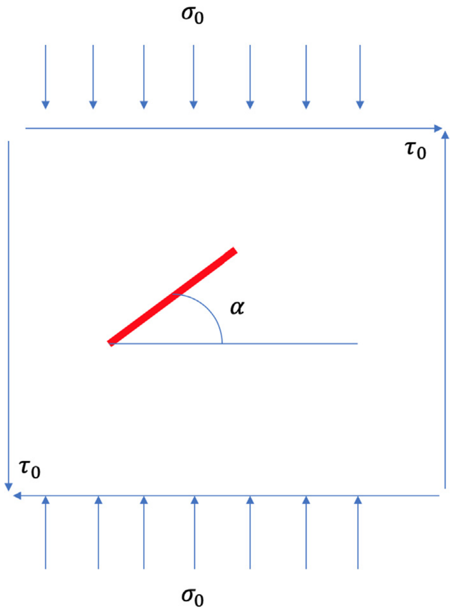

Fig. 1. Inclined crack in a plane loaded by a combination of compressive and shearing load applied at infinity.

Thus, the possibility of even a small (as a percentage) reduction of the energy consumption of the rock fracture process may have a huge economic effect. The present paper is primarily focused on the analysis of energy input required for initiation of existing crack (see ex. [19,17]) in a quasibrittle media loaded by uniaxial compressive loading and combination of compressive and shear loading applied at a large distance from the crack.

2. Problem statement

Since the study is primarily aimed at fracture and fragmentation of rocks, which, in most cases, are quasibrittle heterogenous materials, as the simplest approximation, the material behavior and strength can be described using linear elasticity and linear elastic fracture mechanics (LEFM). Equilibrium equations are written as Lame equations in the absence of mass forces and inertial terms (in the approximation of a quasistatic problem):

$$(\lambda + \mu)u_{j,ji} + \mu u_{i,jj} = 0, \tag{1}$$

where u are displacements in the plane in directions 1 и 2, λ and μ are the Lame coefficients, i and j are taking values 1 and 2 and comma stands for partial derivative by the index following this comma. Einstein summation convention is adopted. Stresses are coupled with strains by the Hooke's law:

$$\sigma_{ij} = \lambda\delta_{ij}u_{k,k} + \mu(u_{i,j} + u_{j,i}), \tag{2}$$

where σ_{ij} are the stress tensor components and δ_{ij} is the delta function taking the value of 1 when $i = j$ and taking the value of 0 in all the other cases.

Geometry of the problem is presented in Fig. 1.

As follows from Fig. 1, in an infinite plane there is a crack of length $2a$ located at an angle of $\frac{\pi}{4} - \alpha$ to the direction of compressive stresses having an intensity (amplitude) of σ_0. Also, at infinity shear stresses are applied having an intensity of τ_0.

As the first step, we find what combinations of compressive stresses σ_0 and shear stresses τ_0 can result in fracture in the crack tips. Utilizing the known formulas for stress tensor transformation, one can find stresses in the loaded plane (not accounting for the stresses created due to the presence of the crack) for the coordinate system with one of the axes coinciding with the direction of the crack:

$$\begin{aligned} \sigma_t &= \sigma_0 sin^2(\alpha) + 2\tau_0 sin(\alpha)\cos(\alpha) \\ \sigma_n &= \sigma_0 cos^2(\alpha) - 2\tau_0 sin(\alpha)\cos(\alpha) \\ \tau_{nt} &= -\sigma_0 sin(\alpha)cos(\alpha) + \tau_0(cos^2(\alpha) - sin^2(\alpha)), \end{aligned} \tag{3}$$

where the direction n is the direction aligned with the crack and the direction t is normal to the direction of the crack.

Now it is possible to find stress intensity factors appearing in the regions surrounding the crack tips:

$$\begin{aligned} K_I &= \sigma_t\sqrt{\pi a} \\ K_{II} &= \tau_{nt}\sqrt{\pi a}. \end{aligned} \tag{4}$$

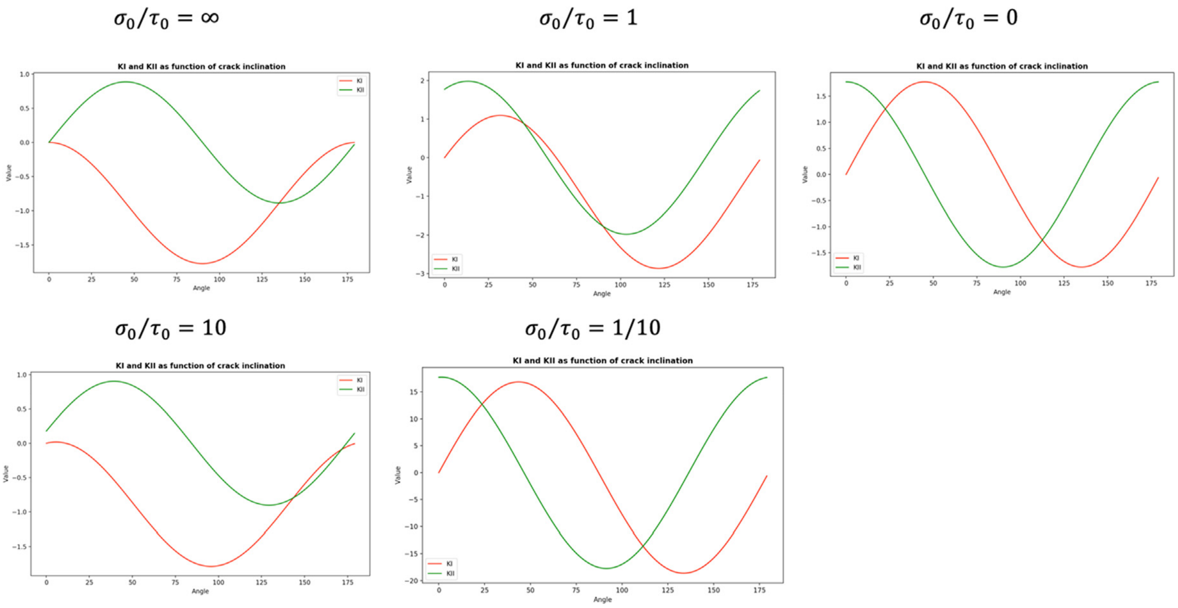

Fig. 2. Stress intensity factors for the first mode K_I and the second mode K_{II} as a function of the crack orientation for different σ_0/τ_0 ratio.

Substituting (3) into (4) one arrives at equations for crack tip stress intensity factors for the first (K_{I}) and the second (K_{II}) mode, appearing for the problem solved. In Fig. 2 one can see the examples of the calculated stress intensity factors as a function of the crack orientation angle α for various combinations of compression stresses σ_0 and tensile stresses τ_0 applied at infinity. Notable are limiting situations: $\sigma_0/\tau_0 = \infty$ corresponds to the situation with no tensile load applied – fracture can appear only as a result of the action of the compressive load and $\sigma_0/\tau_0 = 0$, when no compressive load is applied and fracture can be the result of the shear loading only. Other situations correspond to different combinations of compression and shear stresses with different ratios. For the approximation used, the action of crack faces on each other is neglected: i.e. crack faces can overlap, which results in negative stress intensity factor K_{I} appearing in the crack tip.

Introducing polar coordinate system with the pole at the crack tip (see Fig. 3), one can explicitly present stresses in the vicinity of the studied crack tip:

$$\begin{aligned}
\sigma_r &= \frac{1}{2\sqrt{2\pi r}}\left[K_I(3-\cos\theta)cos\frac{\theta}{2} + K_{II}(3\cos\theta - 1)sin\frac{\theta}{2}\right] + \sigma_n sin^2\theta \\
\sigma_\theta &= \frac{1}{2\sqrt{2\pi r}}cos\frac{\theta}{2}[K_I(1+\cos\theta) - 3K_{II}sin\theta] + \sigma_n cos^2\theta \\
\tau_{r\theta} &= \frac{1}{2\sqrt{2\pi r}}cos\frac{\theta}{2}[K_I\sin\theta + K_{II}(3\cos\theta - 1)] - \sigma_n\sin\theta\cos\theta
\end{aligned} \tag{5}$$

In order to analyze the amplitudes of the loads leading to fracture in the crack tip, a fracture criterion should be introduced. For example, in [10] the most commonly used fracture criteria in the case of mixed bidirectional loading are presented. Some of these approaches are based on the magnitudes of the stress intensity factors. For another group of fracture models, intensities of the local stress fields are utilized as the main parameter. Alternatively, a number of studies rely on energy reasoning. Discussion of different fracture criteria is not within the scope of the current paper. Usage of different fracture criteria will result in slightly different, but still qualitatively very similar results in terms of energy required for initiation of the studied crack. For the further analysis we will adopt a fracture criterion proposed by Neuber and Novozhilov [11,12]. A similar approach is used in [1], where it is shown that estimations

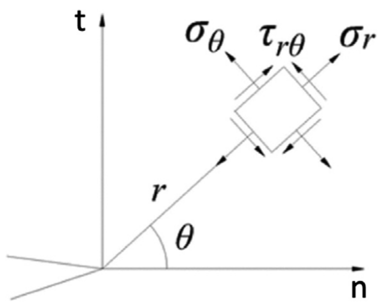

Fig. 3. Polar coordinate system associated with the tip of the crack.

received utilizing the described approach are in the best coincidence with the results for critical loads leading to crack initiation and for direction of crack propagation observed in real experimental conditions. Neuber-Novoshilov criterion can be written as (see ex. [14]):

$$\frac{1}{d}\int_0^d \sigma_\theta(r)dr \geq \sigma_c, \tag{6}$$

where σ_c is the critical tensile stress (ultimate stress), typical for fractured material in the case of loading of a sample without macroscopic defects. Lower integration limit corresponds to the point where rupture is assessed (in our case that is the crack tip). Spatial value d is chosen from the condition of correspondence of criterion (6) to Griffith-Irwine [20,21] criterion of critical stress intensity factor in the case of square-root singular asymptotic controlling the stress field (see ex. [15]): $d = \frac{2}{\pi}\left(\frac{K_{IC}}{\sigma_c}\right)^2$, where $K_{IC}$is the critical stress intensity factor, typical for the fractured material.

Applying the fracture criterion (6) to the analysis of rupture in the case of the stress field given by (5), one will receive:

$$\bar{\sigma}_\theta = \frac{1}{d}\int_0^d \sigma_\theta(r)dr = \sqrt{\frac{2}{\pi d}}\cos\frac{\theta}{2}\left[K_I\cos^2\frac{\theta}{2} - \frac{3}{2}K_{II}\sin\theta\right] + \sigma_n\sin^2\theta \tag{7}$$

Direction of crack propagation can also be found as the direction for which the fracture condition is executed for the lowest fracturing load amplitude:

$$\frac{\partial\overline{\sigma_\theta}}{\partial\theta} = 0, \frac{\partial^2\overline{\sigma_\theta}}{\partial\theta^2} < 0$$

Or, substituting (7) into the equation above:

$$\frac{\partial\overline{\sigma_\theta}}{\partial\theta} = K_I\sin\theta_0 + K_{II}(3\cos\theta_0 - 1) - \frac{8\sigma_n}{3}\sqrt{2\pi d}\cos\theta_0\sin\frac{\theta_0}{2} = 0, \tag{8}$$

where θ_0 will give the direction of the predicted crack propagation. The results received for critical load amplitude and for crack propagation direction can be compared to the ones observed experimentally (ex. see [1]). The computed crack initiation angle as a function of initial crack inclination for different ratios of compressive and shearing loads is presented in Fig. 4. Discontinuities in initiation angle denoted by vertical dashed line are due to initiation angle hitting 360 degrees.

Should the material properties be defined (including the ultimate stress and the critical stress intensity factor), utilizing formula (7), for every σ_0/τ_0 ratio one can find the minimum load amplitude for which the crack should be initiated at the crack tip. After that, it is possible to evaluate minimum specific (per unit volume) energy, that should be transferred to the plane with the crack in order to create conditions needed for crack initiation:

$$\mathrm{E} = \sigma_{ij}\varepsilon_{ij}dV = \left(\frac{\sigma_0^2}{E} + \frac{\tau_0^2}{G}\right)dV. \tag{9}$$

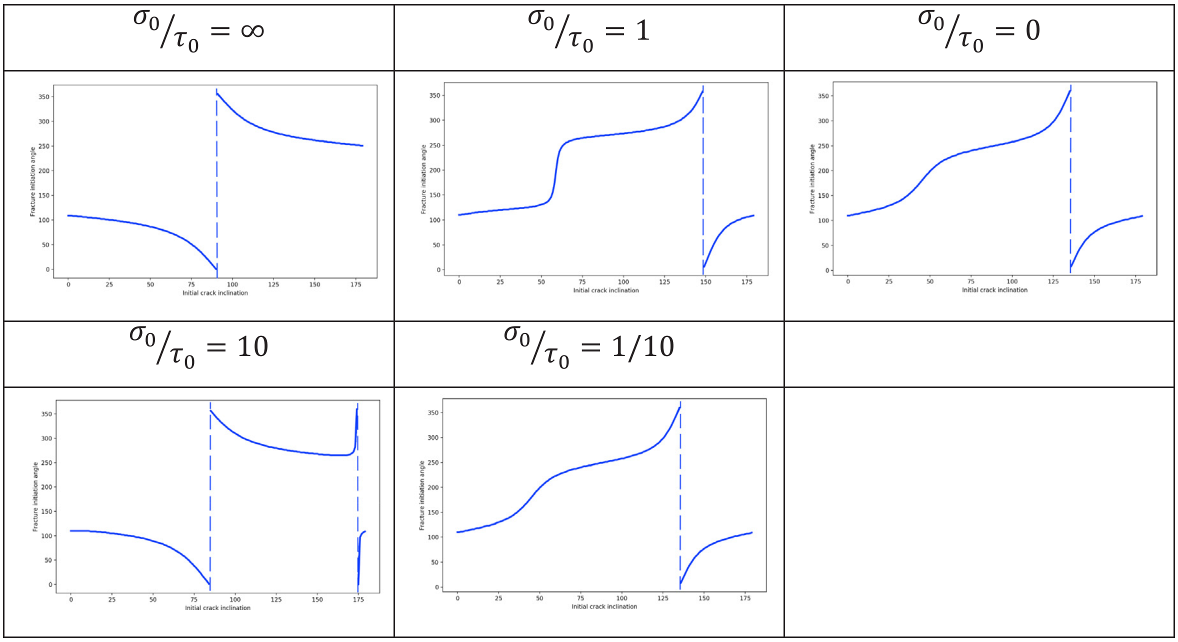

Fig. 4. Crack initiation angle as a function of initial crack inclination for different σ_0/τ_0 ratios.

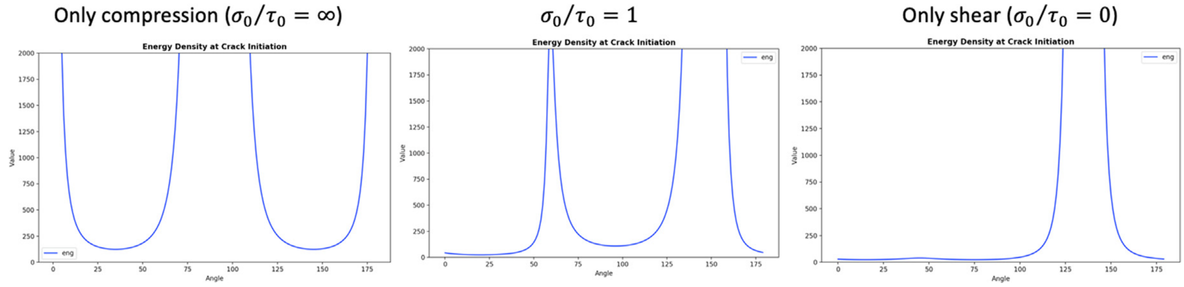

Fig. 5. Minimum specific energy (J/m^2), required for initiation of fracture as a function of crack orientation for different σ_0/τ_0 ratios.

This specific energy will be proportional to the minimum input of energy necessary for creation of conditions required for growth of existing crack within the bulk of the studied material.

The developed approach can be applied for analysis of fracture of a real material. As a model material we choose granite with the following properties: Young's modulus E = 50 GPa, shear modulus G = 24e9 Pa, critical tensile stress (ultimate stress) σ_C= 20e6 Pa, critical stress intensity factor $K_{IC} = 1.7\text{e}6\ \ \text{Pa}\sqrt{m}$. The received results should qualitatively hold for a wide range of different rock materials. Fig. 5 shows the computed dependencies for minimum specific energy, that should be transferred to the material in order to create fracture in the tip of the existing inclined crack. This specific energy is plotted as a function of orientation of the existing crack for various σ_0/τ_0 ratios.

Fig. 6 gives minimum specific energy, required for initiation of fracture as a function of the ratio τ_0/σ_0 between the intensities of the compressive and the shear loading.

Minimum specific energy is an important characteristic related to minimum possible energy required to initiate a crack with most favorable orientation. At the same time, for real processes single impact of a fracturing machine should produce fracture in existing cracks with a spectrum of different orientations. Thus, another characteristic can be introduced – specific energy required for initiation of 30% of existing cracks (supposing uniform distribution of crack orientations). This value will be related to energy needed for incrementation of 30% of cracks existing within the bulk of the fractured media. Fig. 7 plots this specific energy as a function of the τ_0/σ_0 ratio.

3. Conclusions

As evident from Figs. 6 and 7, an attempt to initiate fracture in the tip of the existing crack utilizing purely compressive loading is the least energetically advantageous from all of the considered loading possibilities. Addition of shearing component to the applied load results in significant reduction of energy required for initiation of fracture in the tip of the existing crack. For ratio $\tau_0/\sigma_0 \approx 1$ (similar amplitude of the compressive and the shearing load) the required energy is reduced almost by the factor of 6. Further

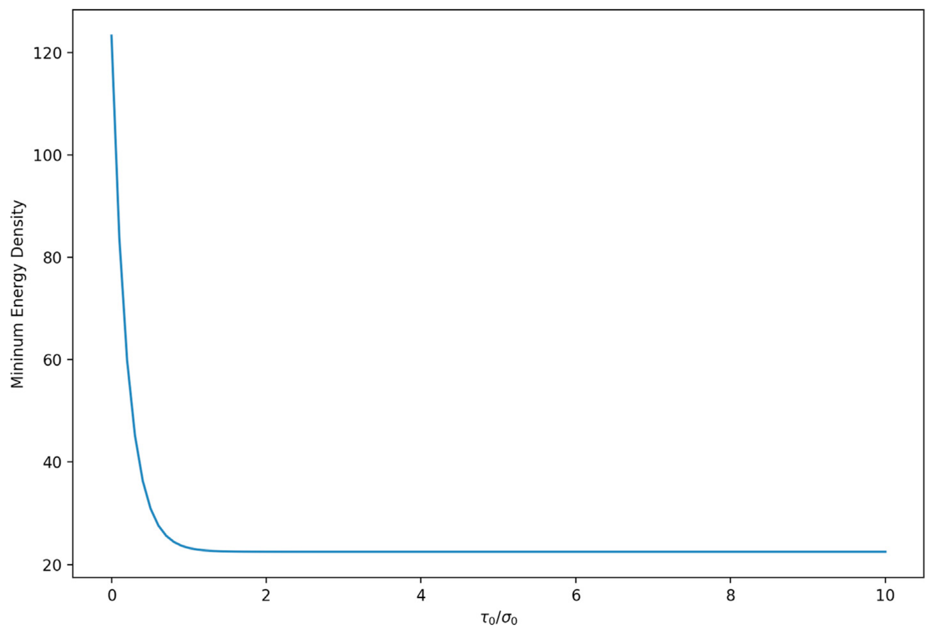

Fig. 6. Minimum specific energy, required for initiation of fracture as a function of the τ_0/σ_0 ratio.

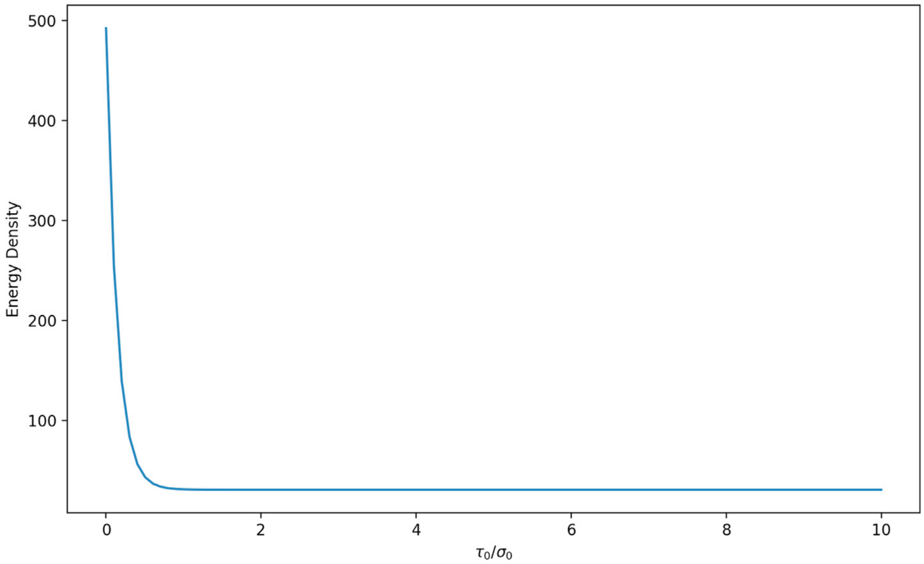

Fig. 7. Specific energy, required for initiation of fracture in 30% of cracks (supposing uniform distribution of crack directions) as a function of the τ_0/σ_0 ratio.

increase of the amplitude of the shearing load is not yielding the significant reduction of minimum energy required for creation of fracture in the crack tip.

The received results are qualitatively coinciding with effects observed in reality. It was shown experimentally [16,17], that addition of sharing component to the applied loading results in substantial decrease of energy consumed by the processes of grinding and fragmentation of rocks and other quasibrittle materials. The results presented in this paper can be applied for prediction of optimal energy saving parameters for industrial devices working for grinding and fragmentation of quasibrittle materials.

The demonstrated effect is received in quasistatic conditions for a single crack in an infinite plane. Real rock materials contain numerous cracks with distribution of dimensions and orientations that can be studied experimentally (for ex. using X-ray computer tomography). It is also an open question if dynamic (inertia) effects do influence fracture and strength of industrially processed rock materials in a significant way. Future research in the area should include both the analysis of array of cracks with different dimensions and orientations as well as the study of dynamic problem accounting for wave propagation and effects of inertia.

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