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HOMOGENEOUS HORIZONTAL AND VERTICAL SEISMIC BARRIERS: MATHEMATICAL FOUNDATIONS AND DIMENSIONAL ANALYSIS

V.A. Bratov[1,2*], A.V. Ilyashenko[3], S.V. Kuznetsov[4], T.-K. Lin[5], N.F. Morozov[2]

[1]Institute for Problems in Mechanical Engineering RAS, Bolshoj pr. 61, St Petersburg, 199178, Russia

[2]St Petersburg State University, 13B Universitetskaya Emb, St Petersburg 199034, Russia

[3]Moscow State University of Civil Engineering, Yaroslavsckoe sh. 26, Moscow, Russia

[4]Ishlinsky Institute for Problems in Mechanics RAS, Prospekt Vernadskogo 101-1, Moscow 119526, Russia

[5]National Chiao Tung University, Hsinchu, Taiwan, 300

*e-mail: vladimir@bratov.com

Abstract. The concept of a vertical barrier embedded in soil to protect from seismic waves of the Rayleigh type is discussed. Horizontal barriers are also analyzed. The principle idea for such a barrier is to reflect and scatter energy of an oncoming wave by the barrier, thus decreasing the amplitude of surface vibrations beyond the barrier. Numerical FE simulations of a plane model are presented and discussed.

Keywords: seismic protection, seismic barrier, Rayleigh waves, Lamb problem

1. Introduction

Ground vibrations generated by the external sources, such as earthquakes, blasts, railroads, etc. can affect structures and cause their damage. During recent few decades, several approaches were suggested to mitigate effects of the ground vibrations inside the protected regions by introducing barriers of different nature; see [1-11].

Most of these works concern with vertical barriers filled by an acoustically softer material than the one of the ambient soil. However, as observed in [8], horizontal barriers filled by acoustically stiffer material than the ambient soil can produce even stronger protective effect against vibrations. The discussed effect relates to Chadwick's theorem [12,13] stating that no Rayleigh waves can propagate over a clamped surface of a halfspace or a halfplane.

Herein, different materials for filling in the vertical barriers are analyzed with respect to their ability to mitigate ground vibrations beyond the barrier. The main attention is paid to Rayleigh waves, as the major factor causing ground surface vibrations at regions sufficiently distant from the buried dynamic sources [14].

2. Basic notations

The starting point for the analysis of interaction of surface acoustic waves (in the considered case of a homogeneous halfplane the surface waves are reduced to Rayleigh waves) with the vertical barrier, is analysis of the equation of motion

$$c_P \nabla \operatorname{div} \mathbf{u} - c_S \operatorname{rot} \operatorname{rot} \mathbf{u} = \frac{\partial^2 \mathbf{u}}{\partial t^2} , \tag{1}$$

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where $\mathbf{u}$ is the displacement field, c_P and c_S are velocities of the longitudinal and transverse bulk waves respectively:

$$c_P = \sqrt{\frac{\lambda+2\mu}{\rho}}, \quad c_S = \sqrt{\frac{\mu}{\rho}}. \tag{2}$$

In (2) λ and μ are Lamé constants and ρ is the material density.

Due to Helmholtz decomposition, the displacement field can be represented in terms of scalar (Φ) and vector ($\boldsymbol{\Psi}$) potentials

$$\mathbf{u} = \nabla\Phi + \operatorname{rot}\boldsymbol{\Psi}. \tag{3}$$

The potentials are assumed harmonic in time

$$\Phi(\mathbf{x},t) = \Phi'(\mathbf{x})e^{i\omega t}, \quad \boldsymbol{\Psi}(\mathbf{x},t) = \boldsymbol{\Psi}'(\mathbf{x})e^{i\omega t}. \tag{4}$$

Substituting representation (4) into Eq. (1) yields two independent Helmholtz equations

$$\left(\Delta + \frac{\omega^2}{c_P^2}\right)\Phi' = 0, \quad \left(\Delta + \frac{\omega^2}{c_S^2}\right)\boldsymbol{\Psi}' = 0. \tag{5}$$

To define plane waves and to simplify the analysis, the splitting spatial argument is needed

$$\mathbf{x} = (\mathbf{x}\cdot\mathbf{n})\mathbf{n} + (\mathbf{x}\cdot\boldsymbol{\nu})\boldsymbol{\nu} + (\mathbf{x}\cdot\mathbf{w})\mathbf{w}, \tag{6}$$

where $\mathbf{n}$ is the unit wave vector, is the unit normal to the median plane of the plate, and $\mathbf{w} = \mathbf{n}\times\boldsymbol{\nu}$.

The further assumption relates to the periodicity of the potentials in the direction of propagation

$$\Phi'(\mathbf{x}) = \varphi(x'')e^{x'}, \quad \Psi'(\mathbf{x}) = \psi(x'')e^{x'}, \tag{7}$$

where the dimensionless complex coordinates x' and x'' are

$$x' = ir\,\mathbf{x}\cdot\mathbf{n}, \quad x'' = ir\,\mathbf{x}\cdot\boldsymbol{\nu}. \tag{8}$$

In (8) $i = \sqrt{-1}$ and r is the wave number related to the wavelength l by

$$r = \frac{2\pi}{l}. \tag{9}$$

Substituting representations (7) into Eq. (5) results in the decoupled system of two ordinary differential equations

$$\frac{d^2\varphi}{dx''^2} + \left(1 - \frac{c^2}{c_P^2}\right)\varphi = 0, \quad \frac{d^2\psi}{dx''^2} + \left(1 - \frac{c^2}{c_S^2}\right)\psi = 0, \tag{10}$$

where the phase speed c relates to the frequency and the wave number by the following relation

$$c = \frac{\omega}{r}. \tag{11}$$

The boundary surface is $\mathbf{x}\cdot\boldsymbol{\nu} = 0$ assumed free from the surface tractions:

$$\mathbf{t}_\nu \equiv \left(\lambda\,\mathrm{tr}(\nabla\mathbf{u})\mathbf{I} + \mu\left(\nabla\mathbf{u} + \nabla\mathbf{u}^t\right)\right)\cdot\boldsymbol{\nu} = 0, \quad \mathbf{x}\cdot\boldsymbol{\nu} = 0. \tag{12}$$

Substitution representation (3) into boundary conditions (12) yields boundary conditions written in terms of potentials φ'' and $\boldsymbol{\psi}''$

$$\left(\lambda\Delta\Phi'\mathbf{I} + 2\mu\left(\nabla\nabla\Phi' + \tfrac{1}{2}\left(\nabla\operatorname{rot}\boldsymbol{\Psi}' + (\nabla\operatorname{rot}\boldsymbol{\Psi}')^t\right)\right)\right)\cdot\boldsymbol{\nu} = 0, \quad \mathbf{x}\cdot\boldsymbol{\nu} = 0. \tag{13}$$

The equation (13) is one, we are looking for; it describes propagation of Rayleigh waves along free surface of a halfspace/halfplane.

Equation (13) should be supplemented with equation of motion for the barrier, analogous to Eq. (1), and boundary conditions at the interface between barrier and soil. The ideal mechanical contact is imposed at the interface:

$$\left.\begin{aligned} \mathbf{u}_{bar} &= \mathbf{u}_{soil} \\ \mathbf{t}_{\nu_{bar}} &= \mathbf{t}_{\nu_{soil}} \end{aligned}\right|_{\text{interface}} \tag{14}$$

In the next section the FE approach for solving the considered equations will be developed, allowing us to analyze the interaction of Rayleigh waves with the vertical seismic barrier.

3. FE modeling of a system "soil-vertical barrier"

Herein, some results based on numerical modeling of seismic waves propagation as well as their interaction with vertical seismic barriers are presented. The shown results are received utilizing an explicit FE code.

Basic Remarks. The analysis has shown that similarly to the horizontal barriers [8], vertical barriers should satisfy several important conditions in order to protect the given area from seismic waves effectively: (i) height of the barrier should be comparable with the lengths of the waves which it protects from; (ii) material of the barrier should have larger Young's module and density than the ambient soil has (iii).

2D Model. In connection with the complexity of this problem, 2D model was used in order to simplify the subsequent studies. These are models consisting of a symmetric plate with sizes which were chosen lest the waves reflected from the boundaries of the model should return to the points of observation during the calculation time. The condition of symmetry (3) is applied on the left edge of the plate while, the lower and the right edges were fixed. The source of waves was simulated as a harmonic load (1) applied on the upper edge in the center of the plate (on the top of the axe of symmetry). Vertical barrier (2) was created at a distance from the axe of symmetry so that the wave picture might stabilize. Figure 1 represents the picture of wave propagation in the model.

Comparing the kinetic energy of a piece of the plate beyond the barrier with the energy of the same area without barrier provides us with the information on the efficiency of this barrier. The same comparison may be carried out with the magnitudes of displacement of the observation points behind the barrier.

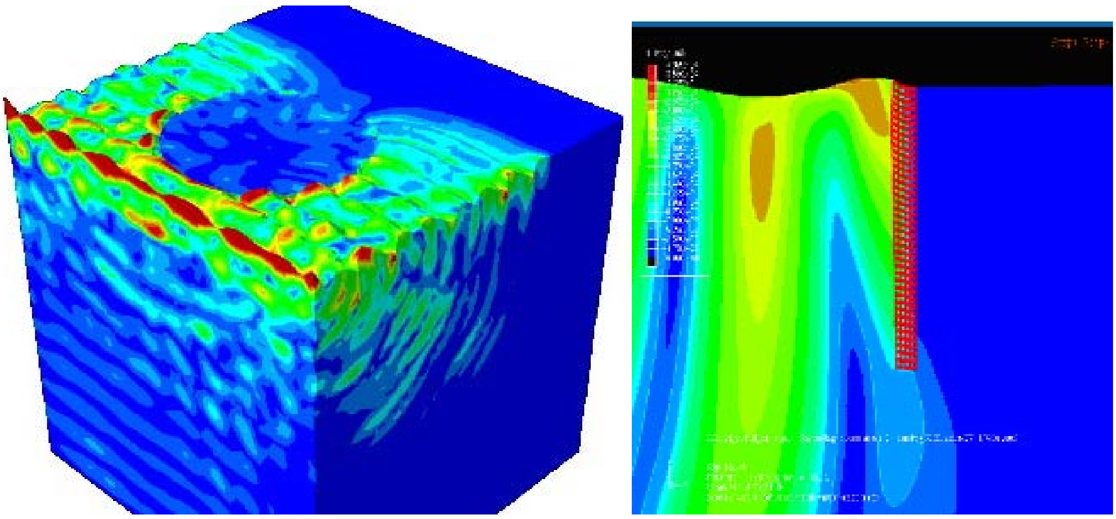

Fig. 1. Finite element model with a vertical round-shaped barrier. 3D model (left) and cross section (right)

Similarly, Figure 2 shows a finite element model of a horizontal barrier. The latter utilizes Chadwick's theorem on non-propagating Rayleigh waves in a clamped halfspace. In view of this theorem, the modeled horizontal barrier had either larger Young's modulus than the halfspace, or larger density, or both. The latter case, as numerical computations reveal, appears the best in terms of reduction vibrations behind the barrier: Rayleigh waves are almost completely eliminated in the protecting zone.

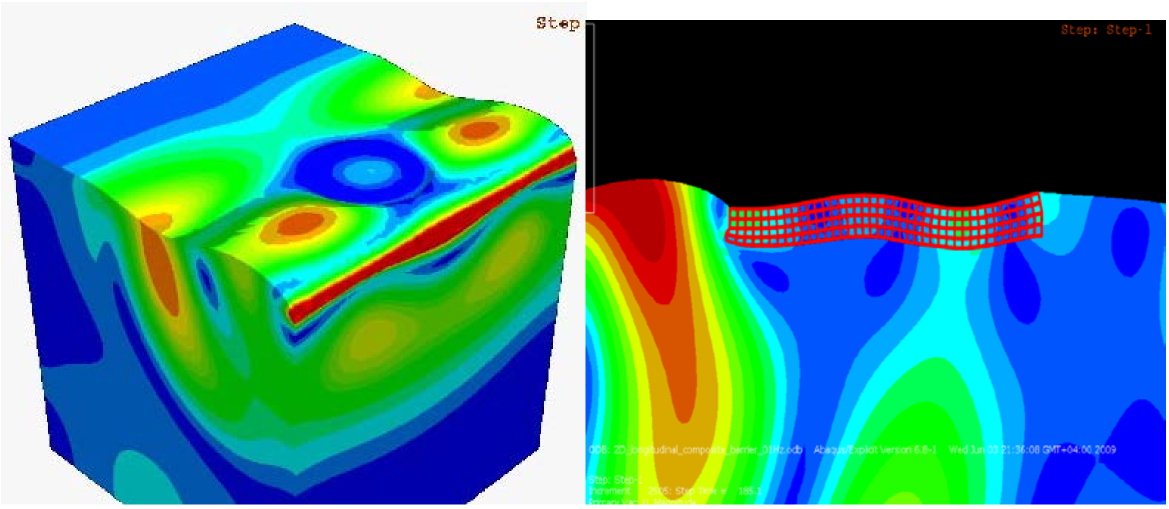

Fig. 2. Finite element model with a horizontal round-shaped barrier. 3D model (left) and cross section (right)

4. Dimensional analysis

In accordance with the π-theorem [15] which states that physical law does not depend on the form of units, the kinetic energy field E_{kin}^{bar} of an area Δ beyond the barrier can be described by the following group of dimensionless parameters:

$$E_{kin}^{bar}\left(\frac{E_{bar}}{E_{soil}};\frac{\rho_{bar}}{\rho_{soil}};\frac{d\times h}{\lambda^2};\frac{d}{h};\frac{\Delta}{\lambda};\frac{\omega\lambda_{soil}}{\sqrt{E_{soil}/\rho_{soil}}}\nu_{bar};\nu_{soil}\right), \tag{15}$$

where index *soil* marks the ambient material of the half-space, while index *bar* corresponds to the parameters of the barrier; λ is the wavelength of the Rayleigh wave in a half-space (this wavelength can be solved from the Bergmann-Victorov's equation); E_{bar}, E_{soil} are the corresponding Young's moduli; ν_{bar}, ν_{soil} are the Poisson's ratios; ρ_{bar}, ρ_{soil} are the densities; d and h are the thickness and the height of the barrier accordingly; ω is the circular frequency of the exciting load (here it is always equal to the wave circular frequency).

According to the analyses performed in [8] as well as this research, both Poisson's ratios almost do not have the influence on the kinetic energy field of the area, therefore, we can eliminate both Poisson's ratios. Apart from that, the frequencies of considered waves remain constant (because the applied harmonic load has a constant frequency). That is why the expression (15) can be simplified to the following:

$$E_{kin}^{bar}\left(\frac{E_{bar}}{E_{soil}};\frac{\rho_{bar}}{\rho_{soil}};\frac{d\times h}{\lambda^2};\frac{d}{h}\right). \tag{16}$$

5. Conclusions

It was demonstrated that seismic barriers can be utilized to successfully protect areas from oncoming seismic waves significantly reducing amplitudes of vibrations and surface accelerations. Further research is required in order to establish best possible geometry and material for the barriers. The developed approach can be verified experimentally on both the laboratory scale and outdoor experiment.

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